Pricing Disaster Risk in Corporate Bonds

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Abstract

Realized default rates and losses are too low relative to high corporate credit spreads in the data. I explain this "credit spread puzzle" through a dynamic capital structure model with long-term bonds and disaster risk. In contrast to models with one-period bonds, long-term bonds serve as hedge investments under disaster risk concerns, generating different economic mechanisms. I explore the mechanisms at firm level. Disaster risk affects corporate credit spreads through default risk, the risk premium, and the corporate capital structure. In disaster states, default risk dominates other channels due to firms' high likelihood of default. In normal times, the disaster risk premium induces lower optimal capital levels, which leads to higher leverage and credit spreads. When exploring the effect of real and financing frictions in the model, I find that both lead firms to act conservatively and reduce their leverage, giving rise to lower credit spreads. However, I also find that, after a disaster, financially constrained firms lose more equity value, and their credit spreads sharply increase in the model.

JEL Codes: E22, E32, E44, G12, G32, G33, C61

Keywords: Disaster risk, credit spreads, real investment, dynamic capital structure

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1 Introduction

How does disaster risk affect corporate credit spreads? As shown in Figure 1, the Baa-Aaa credit spread reaches as high as 6%, and the annual default rate peaks at 8% during the Great Depression. Disasters such as the Great Depression are associated with large negative productivity shocks, which tend to substantially increase default probability and loss. Moreover, during or even before the realization of disasters, investor fears drives up the risk premium. I formalize these ideas to demonstrate that disaster risk has a first-order impact on corporate bond prices and credit spreads. Because disasters are rare events and endogenously change both investment and financing decisions, it is challenging to empirically study these impacts. Therefore, I approach this problem by providing a dynamic quantitative model of firms that features disaster risk.

Specifically, in this paper, I introduce disaster risk into a capital structure model with long-term defaultable bonds. A stochastic discount factor based on the Epstein-Zin utility incorporates disaster risk, with a small chance of a consumption growth crash. Firms are obligated to repay a predetermined amount of coupon and principal each period and can choose to default when the continuation value is negative. The defaultable bonds are priced based on future expected default probability and recovery rate. Firms make endogenous forward-looking investment decisions, which are financed by both equity and bonds. Firms face investment adjustment costs, and fixed and proportional costs when issuing new equity and bonds.

There are two main drivers in the model that affect credit spreads. First, as in other bond pricing models, countercyclical variations in the risk premium, default risk, and default loss drive up credit spreads. In bad times, when investors' marginal utility is high and they require a higher rate of return, firms face higher default risk and lower continuation value, leading to increased credit spreads. Disaster risk exaggerates these effects by adding significantly higher default losses and probabilities in the disaster state, and introduces a highly nonlinear pricing kernel. The risk premium channel is critical in normal times, while the default risk channel plays a larger role in realized disaster states.

In addition to the risk premium and default risk channels, firms change their optimal capital structures when facing rare but severe disaster risks. During normal times with the threat of disaster risk, firms choose lower optimal capital levels. Lower capital levels lead to lower equity value and more volatile investments because firms have to sell capital in bad times. Lower capital levels also lead to higher market leverage, generating high credit spreads.

Why do firms not lower bond levels to decrease leverage and avoid default in disaster states? On the one hand, long-term bonds are attractive to firms, primarily due to the rapid recovery period following the disaster state.[1] In each period, firms only have to repay a portion of the principal outstanding, providing less liquidity pressure when a disaster hits and more time for the firm to recover in the subsequent recovery state. On the other hand, with less capital, firms are forced to issue new bonds to obtain sufficient liquidity and repay the promised amount of principal. Firms are also more likely to roll over the bonds or issue new equity when the real and financial frictions are not too high. The incentive to issue bonds outweighs the fear of immediate disaster realization and leads to higher average credit spreads in equilibrium.

A model with long-term defaultable bonds is computationally challenging to solve, because it does not satisfy the contraction mapping theorem, and algorithm convergence is not guaranteed. Therefore, I combine the strategies in Chatterjee and Eyigungor (2012) and Kuehn and Schmid (2014). One problem is that there are multiple local optima, generating oscillations in the algorithm iterations. Following Chatterjee and Eyigungor, I introduce a small transitory income shock to solve it. Another issue is the impact of the initial value. I follow Kuehn and Schmid and use the model solution for shorter-term bonds as the initial value for longer-term bonds. I start with one-period bonds and gradually achieve five-year maturity. The third issue is the kink in the value function and the numerical errors generated in interpolations. I interpolate on the relatively

[1] Giglio et al. (2021) discuss the difference between two types of disaster models. The first is that economic and consumption activity causes disaster risk. The second is that disaster risk drives consumption growth. After a disaster hits, the economy adapts and recovers from the disaster. A firm with a long-term bond adapts to the disaster better than a firm with a one-period bond because it has a longer time to repay the full principal. Bai et al. (2019) also note that long-term bonds have a lower yield for similar reasons.

smoother continuation value function to reduce numerical errors.

To use the model to study the counterfactual effects in disaster states, careful calibration is critical. I simulate the economy and match the key firm-level moments in samples without realized disasters to empirical observations in US data. On the representative consumer side, parameter values in the Epstein-Zin utility function reproduce moments that match aggregate consumption growth data and the equity risk premium. On the firm side, parameter values generate moments that match firm-level default rates, credit spreads, and investment and financing policies.

Using the calibrated model, I focus on three main questions: How does disaster risk affect corporate credit spreads? What are the implications at the firm level in normal times and disaster states? What are the counterfactual results when firms face higher or lower real and financial frictions?

Disaster risk has a first-order impact on credit spreads. Consistent with empirical observations in normal times, my model generates an average credit spread of 92.79 basis points (bps), close to the 70–100 bps in Duffee (1998). Moreover, my model reproduces an annual default rate of 0.66%, an annual equity premium of 9.09%, and an average market leverage of 31%. The empirical benchmark of the annual default rate, 0.7%, is from Moody's (2013) annual report on corporate default and recovery rates. The annual equity premium is 7.06% the mean of the annual excess return from 1872 to 2021. The empirical market leverage from the Compustat Quarterly data is 24% from 1983 to 2022. In Fan et al. (2012), the mean market leverage, 29%, is estimated using a cross-country panel.

As a comparative static, a model with a less severe consumption crash or lower disaster probability generates an average credit spread of 33.96 bps and 30.36 bps, respectively. With a less severe consumption crash or a lower disaster probability, there is a lower expected default probability and risk premium. Firms are able to accumulate capital and increase equity value, leading to lower leverage. A lower default risk, risk premium, and leverage lead to decreased credit spreads. Conversely, a model with a more severe consumption crash or higher disaster probability generates higher leverage and an average credit spread of 254.03 bps and 159.68 bps, respectively.

I then analyze the quantitative results when disasters are realized. Intuitively, when a disaster hits, both the default rate and credit spread increase. My model predicts an annual default rate of 10.45% and an average credit spread of 315.15 bps in samples with realized disasters. Compared to normal times, the average credit spread is more than tripled. The credit spread can vary significantly, with a 90% confidence interval between 111.25 bps and 652.73 bps The variation depends on the severity of the disaster and economic condition before entering the disaster state.

During disaster periods, there is a large drop in investment and new financing, along with an increase in market leverage. These results are driven by a significant negative shock to firms' productivity and a sharp decline in the continuation value. The expected future default rate and losses are high, making it very difficult for firms to issue new bonds. As a result, firms are forced to either default or divest to cope with the liquidity shock and repay current liabilities. The expectation of high default rates and leverage leads to high credit spreads during disaster periods.

After exploring how disaster risk affects average credit spreads, I study more effects in the cross-section. I first focus on the firm-specific determinants of credit spreads and then explore firms' heterogeneous responses to realized disasters. Consistent with the prediction from the one-period bond model and previous literature, contemporaneous credit spreads are largely explained by risk premiums, which have the highest R^2, 0.55, in explaining the variation in credit spreads in the cross-section. In contrast, next-period credit spreads are mostly explained by market leverage ($R^2 = 0.27$). The results show that the contemporaneous changes in credit spreads respond more to exogenous productivity shocks. The shock is embedded into endogenous corporate policies, which will be reflected in the credit spreads next period. In contrast, with realized disasters, default risk explains most variations in both contemporaneous and next-period credit spreads. The default risk channel dominates other economic channels when disasters materialize.

Following a realized disaster, firms lose equity value and increase credit spreads, but the effects are heterogeneous. Consistent with Fahlenbrach et al. (2021), firms with more financial flexibility

lose less equity value, and their credit spreads jump less. In the recovery state, the level of credit spreads will converge back to the levels of the pre-disaster period. Similarly, more profitable firms suffer less in equity value losses and their credit spreads stay relatively stable.

What rold do real and financial frictions play? Different from the findings in Kuehn and Schmid (2014), in normal times, more real and financial frictions decrease credit spreads by reducing firms' leverage. When it is more costly to invest or divest, firms choose more conservative corporate policies. Similarly, when the cost of issuing new equity or bonds is high, firms have less flexibility in financing when there is a negative shock. Firms issue fewer bonds and have lower leverage, leading to both lower default rates and credit spreads. Conversely, firms facing fewer real and financial frictions choose more aggressive corporate policies to finance investments. The reason is that it becomes easier to roll over bonds or use new bonds to solve liquidity pressure, overall generating more defaults and higher credit spreads. I also briefly explore other economic mechanisms. Lower recovery rates and higher fixed operational costs also contribute to higher credit spreads because both forces directly reduce repayments when bonds default.

My paper makes three incremental contributions. The first concerns the literature on structural models to quantitatively explain the credit spread puzzle (Merton, 1974; Leland, 1994). Huang and Huang (2012) find that standard structural models fail to generate realistically high credit spreads after being calibrated to observed default rates and losses. At the aggregate level, credit spreads are related to macroeconomic fluctuations (Hackbarth et al., 2006; Chen, 2010; Gourio, 2013; Bai, 2021). At the firm level, Kuehn and Schmid (2014) show that real and financial frictions make corporate bonds risky and generate realistic credit spreads. I study the credit spread puzzle and the effects of real and financial frictions under concerns about disaster risk.

Most structural models use one-period bonds due to computational challenges. In reality, firms issue bonds with substantially longer maturity (Eom et al., 2004; Driessen, 2005; Huang and Huang, 2012; Dickerson et al., 2023). Huang and Huang (2012) calibrate a wide class of structural models to data, showing that the "credit spread puzzle" exists for four-year and ten-year maturity bonds.

Long-term bonds have different default rates and credit spreads from short-term bonds and different implications for firms' capital structure (Gourio and Michaux, 2012; Kuehn and Schmid, 2014). I contribute by proposing a new firm-level capital structure model that incorporates disaster risk and long-term bonds, deepening the understanding of the credit spread puzzle. Additionally, I show that disaster risk directly affects corporate policies, leading to results on real and financial frictions that differ from previous findings.

Second, my paper contributes to the literature on the effects of disaster risk on asset pricing. The existing quantitative literature has primarily focused on using disaster risk to explain the equity premium (Gourio, 2012; Wachter, 2013; Bai et al., 2019), while bond pricing has received relatively less attention. Empirically, Baron et al. (2022) uses a sample of 20 advanced economies from 1870 to 2021 to construct a direct measure of disaster risk. They find that asset prices reflect a disconnect between subjective and objective disaster risk.

Two related papers are Gourio (2013) and Bai (2021). Both study disaster risk and credit risk at the aggregate level with one-period bonds. Gourio (2013) studies the credit risk implications of exogenous disaster risk, without considering real and financial frictions. Bai (2021) examines how endogenous disaster risk arises due to labor market frictions and its impact on aggregate credit risk. In contrast, my work differs by focusing on firm-level credit spreads. I also study more realistic long-term bonds, considering both real and financial frictions. Doing so allows for a more detailed understanding of how disaster risk affects corporate policies and credit spreads at the firm level.

Third, and more broadly, my paper contributes to the financial crisis literature. At the aggregate level, crises, primarily financial crises, raise fundamentally important economic questions because they feature large negative output shocks that cause real damage compared to normal recessions (Claessens and Kose, 2013; Sufi and Taylor, 2022; Stulz, 2023).[2] In the cross-section,

[2]The severe productivity loss during and after crises are nonnegligible (Bordo et al., 2001; Jordà, 2005; Cerra and Saxena, 2008; Reinhart and Rogoff, 2009; Jordà et al., 2013). Crises are triggered by various factors, rational or irrational (see Sufi and Taylor, 2022 for a comprehensive review). The most important factors are credit expansion, asset price growth, and their interaction (Schularick and Taylor, 2012; Jordà et al., 2015; Mian et al., 2017; Baron et al., 2021; Adrian et al., 2022; Greenwood et al., 2022). In this paper, I do not focus on the causes of crises. I use

firms with greater financial flexibility handle the shocks better, reflected in less loss in stock value (Fahlenbrach et al., 2021). Following models in the disaster risk literature, which directly model potential large output crises, my paper sheds light on how firm-level credit spreads respond to underlying and realized crisis risk through endogenous capital structure choices. Therefore, my paper also contributes to how corporate policies respond to a financial crisis.

The rest of this paper is organized as follows. Section 2 uses a simplified model to illustrate the key economic mechanism. Section 3 describes the full model. Section 4 discusses the calibration strategy. Section 5 reports the quantitative results. Section 6 concludes the paper.

2 An Example with a One-Period Bond

Before delving into the full version of the dynamic model with a long-term bond, I provide an example using a one-period bond to explain the economic mechanisms of the risk premium. Similar to Kuehn and Schmid (2014), the defaultable one-period bond is priced as follows:

$$Q_{it} = \mathrm{E}_t\left[M_{t+1}\left((1-\mathbb{1}^D_{it+1})(1+c)+\mathbb{1}^D_{it+1}R_{it+1}\right)\right] \tag{1}$$

where c is the coupon payment, $\mathbb{1}^D_{it+1}$ is the default indicator, and R_{it+1} is the recovery rate if default occurs. Similarly, the risk-free one-period bond is priced as:

$$Q^f_{it} = (1+c)\mathrm{E}_t[M_{t+1}] \tag{2}$$

The credit spread is the difference between the defaultable and risk-free bond:

$$s_{it} = \frac{1+c}{Q_{it}} - \frac{1+c}{Q^f_{it}} \tag{3}$$

a partial equilibrium model of heterogeneous firms that takes the feature as given to focus on the implications for firms.

Given Equations (1), (2), and (3), the credit spread can be written as:

$$s_{it} = \frac{q_{it} - \chi_{it}}{\mathrm{E}_t[M_{t+1}](1 - q_{it})} \approx q_{it} - \chi_{it} \tag{4}$$

$$\text{in which} \quad q_{it} = \mathrm{E}_t\left[\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}\mathbb{1}^D_{it+1}\right] \tag{5}$$

$$\chi_{it} = \mathrm{E}_t[M_{t+1}\mathbb{1}^D_{it+1}R^*_{it+1}] \tag{6}$$

The credit spread s_{it} increases with the risk-neutral default probability q_{it} and decreases in the value of the recovery rate χ_{it}. The approximation in Equation (4) holds because the risk-free rate $1/\mathrm{E}_t[M_{t+1}]$ is close to one and q_{it} is close to zero. $R^*_{it+1} = R_{it+1}/Q_{it}$ is the recovery rate adjusted by the bond price. If there is no default risk, the wedge between q_{it} and χ_{it} becomes zero, meaning that the credit spread s_{it} is zero.

To more explicitly understand the economic intuition, q_{it} can be decomposed into the actual default probability and the covariance between marginal utility and default risk, while χ_{it} can be decomposed into the expected recovery rate and the covariance between marginal utility and recovery risk. Formally:

$$q_{it} = \mathrm{E}_t[\mathbb{1}^D_{it+1}] + \mathrm{cov}_t\left(\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}, \mathbb{1}^D_{it+1}\right) \tag{7}$$

$$\chi_{it} = \frac{\mathrm{E}_t[R_{it+1}\mathbb{1}^D_{it+1}]}{R_f} + \mathrm{cov}_t\left(M_{t+1}, R_{it+1}\mathbb{1}^D_{it+1}\right) \tag{8}$$

To see the economic mechanisms, consider two scenarios: normal times and disaster states. In normal times, compared to a model with no disaster risk, the expected default probability and loss are higher due to the potential rare but large economic crash. Suppose a disaster hits, firms will experience a large negative shock to productivity and a sudden drop in continuation value, which could trigger more defaults than in normal times. The introduction of disaster risk leads to a highly nonlinear pricing kernel, M_{t+1}, with a stronger covariance with default, $\mathbb{1}^D_{it+1}$, and recovery rates, R_{it+1}, generating a larger risk premium. The combined effect of the expected default risk and risk

premium results in a larger credit spread. However, the real default rate in normal times might be low due to the absence of realized disaster states. Therefore, the inclusion of underlying disaster risk helps explain the credit spread puzzle in normal times, where realized default probability and loss are too low to account for the observed high credit spreads in the data.

In disaster states, the default rates and credit spreads are substantially higher than in normal times. The plummet in firm value is both immediate and persistent. If the economy enters a disaster state, it is expected to remain in that state for several years before entering the recovery state. The persistent low productivity and continuation value lead to more defaults and, as a result, higher credit spreads than in normal times.

However, there is a critical difference between a one-period bond and a long-term bond. When a disaster is realized, a one-period bond defaults immediately because the firm cannot repay the full principal next period, making its continuation value negative. On the other hand, a long-term bond only requires a fraction of the principal repayment and offers a better chance of exploiting the recovery state. Moreover, firms can, in principle, choose their capital structure and investment policies to mitigate the effect of disaster risk on their cost of borrowing. Consequently, uncovering the quantitative effect of disaster risk as a channel for explaining the credit spread puzzle requires a dynamic model with endogenous investment and capital structure. I build such a model in the next section.

3 The Model

I propose a partial equilibrium model with a combination of a standard dynamic corporate finance model (Hennessy and Whited, 2005, 2007; Kuehn and Schmid, 2014) and a standard disaster model (Rietz, 1988; Barro, 2009; Bai et al., 2019). Time is discrete and infinite. Heterogeneous firms face financial frictions when making default, investment, and financing decisions. They encounter both aggregate and idiosyncratic productivity shocks. The representative consumer has Epstein-Zin utility, and the stochastic discount factor is taken by firms as given.

3.1 The Representative Consumer

A representative consumer maximizes Epstein-Zin nonseparable expected utility

$$U_t = \left[(1-\beta)C_t^{1-\frac{1}{\psi}} + \beta\left(E_t[U_{t+1}^{1-\gamma}]\right)^{\frac{1-1/\psi}{1-\gamma}}\right]^{\frac{1}{1-1/\psi}} \tag{9}$$

where β is the time discount factor, γ is the relative risk aversion, and ψ is the elasticity of intertemporal substitution (EIS). The utility function yields the following pricing kernel

$$M_{t+1} = \beta\left(\frac{C_{t+1}}{C_t}\right)^{-\frac{1}{\psi}}\left(\frac{U_{t+1}^{1-\gamma}}{E_t[U_{t+1}^{1-\gamma}]}\right)^{1-\frac{1-1/\psi}{1-\gamma}} \tag{10}$$

Consumption growth is the only aggregate state variable that captures normal business cycles and disasters. Log growth, denoted g_{ct}, follows

$$\log\left(\frac{C_t}{C_{t-1}}\right) \equiv g_{ct} = \bar{g} + g_t \tag{11}$$

where $\bar{g}$ is the average consumption growth rate and g_t is the stochastic shock. Consumption growth has three states: the normal state, disaster state, and recovery state:

$$g_{t+1} = \begin{cases} (1-\eta)g_t^N + \eta\lambda_D & \text{normal state at time } t \\ \theta\lambda_D + (1-\theta)\lambda_R & \text{disaster state at time } t \\ \nu\lambda_R + (1-\nu)g_t^N & \text{recovery state at time } t \end{cases} \tag{12}$$

In normal times, there is a probability η that the economy will enter the disaster state, where consumption growth is $\lambda_D < 0$, representing a large negative shock. If the economy enters the disaster state, the probability that it persists is θ. The probability that it exits and enters the recovery state is $1-\theta$, where consumption growth is $\lambda_R > 0$. If the economy is in the recovery state, it will persist with a probability of ν, or it will return to a normal state. In normal times, consumption growth follows a standard AR(1) process.

$$g_{t+1}^N = \rho_g g_t^N + \sigma_g \epsilon_{t+1} \tag{13}$$

where ϵ_{t+1} follows a standard normal distribution.

Similar to Bai et al. (2019), I first discretize g_t^N into grids that capture normal economic cycles and then add a disaster and a recovery state. I use the Rowenhorst (1995) method to discretize g_t^N into five values, $\{g_i\}_{i=1}^5$, and obtain the transition matrix P_*. Specifically,

$$P_* = \begin{bmatrix} p_{11} & p_{12} & \cdots & p_{15} \\ p_{21} & p_{22} & \cdots & p_{25} \\ \vdots & \vdots & \ddots & \vdots \\ p_{51} & p_{52} & \cdots & p_{55} \end{bmatrix} \tag{14}$$

where $p_{ij} = P_*(g_{t+1} = g_j | g_t = g_i)$, is the transition probability conditional on current consumption growth.

To add the disaster and recovery states, I insert $g_0 = \lambda_D$ and $g_6 = \lambda_R$ into the g_t grid. Here, $\lambda_D < 0$ and $\lambda_R > 0$ capture the size of consumption growth in the disaster and recovery states, respectively. The transition matrix is modified as follows:

$$P = \begin{bmatrix} \theta & 0 & 0 & \dots & 0 & 1-\theta \\ \eta & p_{11}(1-\eta) & p_{12}(1-\eta) & \dots & p_{15}(1-\eta) & 0 \\ \eta & p_{21}(1-\eta) & p_{22}(1-\eta) & \dots & p_{25}(1-\eta) & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ \eta & p_{51}(1-\eta) & p_{52}(1-\eta) & \dots & p_{55}(1-\eta) & 0 \\ 0 & (1-\nu)/5 & (1-\nu)/5 & \dots & (1-\nu)/5 & \nu \end{bmatrix} \tag{15}$$

From any normal state, there is a probability η that the economy enters the disaster state. The persistence of the disaster state is measured by θ, which is the probability of staying in the disaster state after entering. The economy only enters the recovery state following a disaster state, with probability $1-\theta$. The persistence of staying in the recovery state is ν. The probability of entering any of the normal states from the recovery state is $(1-\nu)/5$.

3.2 Heterogeneous Firms

Firms are indexed by i and have production function

$$\Pi_{it} = (1-\tau)(Z_{it} X_t^{1-\alpha} K_{it}^{\alpha} - f K_{it}) \tag{16}$$

where $0 < \alpha < 1$ captures the concavity of production function. X_t is the aggregate productivity, Z_{it} is the firm-level productivity, and K_{it} is capital stock. τ is the tax rate. f is the proportional fixed operating cost.

The growth of aggregate productivity, X_t, is linked to consumption growth. Specifically,

$$\log\left(\frac{X_t}{X_{t-1}}\right) \equiv g_{xt} = \bar{g} + \phi g_t \tag{17}$$

where $\phi > 0$, captures the relative volatility of aggregate productivity growth compared to consumption growth.

The firm-level productivity follows an AR(1) process

$$z_{it+1} = (1-\rho_z)\bar{z} + \rho_z z_{it} + \sigma_z \epsilon^z_{it+1} \tag{18}$$

where $z_{it} = \log(Z_{it})$. ρ_z captures the persistence of the shock and $\bar{z}$ is the unconditional mean. ϵ^z_{it+1} is an independently and identically distributed random variable that follows a standard normal distribution. Specifically, ϵ^z_{it+1} and ϵ^z_{jt+1} are uncorrelated if $i \neq j$. ϵ^z_{it+1} is uncorrelated with aggregate shock ϵ_{t+1} for $\forall i$.

Investment and Adjustment Cost

Firm i chooses investment I_{it} and next-period capital K_{it+1}. Capital accumulates as

$$K_{it+1} = (1-\delta)K_{it} + I_{it} \tag{19}$$

where δ is the depreciation rate. Following Zhang (2005), capital investment incurs asymmetric adjustment costs:

$$\psi(I_{it}, K_{it}) = \begin{cases} \frac{c^+}{2}(\frac{I_{it}}{K_{it}})^2 K_{it}, & \text{for } I_{it} > 0 \\ 0, & \text{for } I_{it} = 0 \\ \frac{c^-}{2}(\frac{I_{it}}{K_{it}})^2 K_{it}, & \text{for } I_{it} < 0 \end{cases} \tag{20}$$

where $c^- > c^+ > 0$, meaning that it is more costly to divest. The costly divestment is critical in bad times when firms attempt to sell capital because of a liquidity shortage.

Costly Financing

Following Kuehn and Schmid (2014), firm i issues long-term bonds to finance investments and dividend distribution. Each period, firms repay κ of the principal outstanding and a fixed coupon c. Thus, the bond has an average maturity of $1/\kappa$ periods. Net bond issuance or repayment is denoted by J_{it}, and the amount of outstanding bonds evolves as:

$$B_{it+1} = (1-\kappa)B_{it} + J_{it} \tag{21}$$

Bond issuance is not frictionless. I assume a positive fixed and proportional cost if bond issuance is positive. When firms repay bonds (negative net bond issuance), there is no cost. Formally, the asymmetric bond adjustment cost follows:

$$\Phi(J_{it}) = (\phi_0 + \phi_1 J_{it})\mathbb{1}_{J_{it}>0} \tag{22}$$

After making investment and bond issuance decisions, firm i generates net cash flow

$$e_{it} = \Pi_{it} - \underbrace{I_{it} - \psi(I_{it}, K_{it})}_{\text{Investment Costs}} + \underbrace{Q_{it}J_{it} - \Phi(J_{it})}_{\text{Bond Issue and Costs}} - \underbrace{((1-\tau)c + \kappa)B_{it}}_{\text{Bond Repayment}} \tag{23}$$

which includes three parts: net output, investment with adjustment cost, and bond issuance and principal repayment. The firm's coupon and principal payments depend on the amount of bond outstanding, B_{it}. The tax shield on bond financing is captured by a tax-deductible coupon payment. As bonds are risky and defaultable, the price Q_{it} is an endogenous function and will be explained in greater detail in the next section.

Positive net cash flow means dividend distribution to shareholders, while negative net cash flow means equity issuance. External equity is costly and includes both a proportional cost, λ_1, and

fixed cost, λ_0. The total equity issuance cost, denoted $\Lambda(e_{it})$, is calculated as follows:

$$\Lambda(e_{it}) = (\lambda_0 + \lambda_1 |e_{it}|)\mathbb{1}_{e_{it}<0} \tag{24}$$

Default and Bond Valuation

At time t, firm i issues the new bond and repays a portion of its outstanding bonds. The price of the new bond, Q_{it}, depends on the expectation of the next-period repayment and recovery rate if the firm defaults. The bond valuation follows:

$$Q_{it} = E_t\left[M_{t+1}\left((1-\mathbb{1}^D_{it+1})(c+\kappa+(1-\kappa)Q_{it+1}) + \mathbb{1}^D_{it+1}\Xi_{it+1}\right)\right] \tag{25}$$

where the recovery rate depends on the next period's productivity and capital level and is bounded by 100%

$$R_{it+1} = \xi \cdot \min\left\{1, \frac{\Pi_{it+1} + (1-\delta)K_{it+1}}{B_{it+1}}\right\} \tag{26}$$

The proportional loss in default is captured by ξ.

Value Maximization

Ultimately, there are four state variables in this model: (1) aggregate productivity X_t, (2) idiosyncratic productivity Z_{it}, (3) capital K_{it}, and (4) bond B_{it}. Let $S(Z_{it}, K_{it}, B_{it}, X_t)$ denote the equity value before the default decision, or the continuation value. When the equity value drops below zero, the firm defaults. The post-default-decision equity value is denoted by

$$V(Z_{it}, K_{it}, B_{it}, X_t) = \max\{0, S(Z_{it}, K_{it}, B_{it}, X_t)\} \tag{27}$$

Given the pricing kernel and realized productivity levels X_t and Z_{it}, firm i chooses the optimal investment, K_{it+1}, and bond, B_{it+1}, to maximize its pre-default equity value:

$$S(Z_{it}, K_{it}, B_{it}, X_t) = \max_{K_{it+1}, B_{it+1}} \left\{e_{it} - \Lambda(e_{it}) + E_t\big[M_{t+1}V(Z_{it+1}, K_{it+1}, B_{it+1}, X_{t+1})\big]\right\} \tag{28}$$

subject to the capital and bond accumulation, and bond pricing equations, the pre-default equity value consists of the current net cash flow and the discounted future post-default equity value.

4 Quantitative Results

In this section, I discuss the quantitative implications of my model, with a focus on credit spreads and default rates. First, I explain the data and how the target moments are measured. Then, I explain my calibration and simulation strategies. Regarding the implications of my model, I first discuss the tables that match the aggregate and firm-level moments and explore how disaster risk affects credit spreads in normal times. Then, I report the counterfactual results regarding how the key variables react when a disaster hits. Last, I study how real and financial frictions affect credit spreads with the threat of disaster risk.

4.1 Data

The firm-level fundamental and financial data are from Compustat's North America Fundamental Quarterly, spanning from 1984 to 2022. Equity market data are from Andrei S. Gonçalves, which is merged between CRSP and Rober Shiller's dataset. The average annual default rate is obtained from Bai (2021). The average five-year default rates and credit spreads are obtained from Kuehn and Schmid (2014). The measurement of cumulative default rate in the simulation follows Moody's (2006).

Investment is measured as the difference between capital expenditure (CAPXY) and the sale of property (SPPEY). Total gross property, plant, and equipment (PPEGTQ) is used to measure capital stock and calculate investment to capital ratio.[3] Equity issuance is the difference between the sale of common and preferred stock (SSTKY) and the purchase of common and preferred stock (PRSTKCY), scaled by lagged total assets (ATQ). The total amount of bonds outstanding includes debt in current liabilities (DLCQ) and long-term debt (DLTTQ). Bond issuance is measured by the

[3] Bai et al. (2022) explains that gross PPE, compared to net PPE, provides a more accurate measure of the investment rate. Net PPE tends to be lower than its economic value, making the investment rate higher than the BEA estimate.

change in total debt outstanding scaled by lagged total assets. All variables that are year-to-date are transformed into quarterly flow variables based on the fiscal year-end.

The market value of equity is calculated as the product of the price per share (PRCCQ) and the number of shares outstanding (CSHOQ). Market leverage is computed by dividing total debt by the sum of total debt and the market value of equity. Consistent with the standard practice in the literature, firms with SIC codes between 4900 and 4999 (utilities industry), 6000 and 6999 (financial industry), or greater than 9000 (other industries) are excluded from the analysis. Additionally, only firms traded on major exchanges, with exchange codes (EXCHG) between 11 and 14, are included in the study.

4.2 Calibration

I calibrate the model at a quarterly frequency. Solving a model with a long-term bond can be numerically challenging and may not guarantee convergence. To address this issue, I employ three strategies to improve convergence. First, I use the solution of the model with a one-period bond as the initial value for the model with a longer-term bond, following the approach in Kuehn and Schmid (2014). During calibration, I solve the model with one-quarter, 2-year, and 4-year bonds. Second, to address oscillation issues arising from multiple local optimal solutions, I incorporate a transitory income shock m_{it} to the net cash flow, Equation (23), as suggested by Chatterjee and Eyigungor (2012). In the net output, $m_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0, 0.001)$ represents the transitory income shock. As shown in Chatterjee and Eyigungor (2012), the i.i.d. transitory income shock helps a model with a long-term bond to converge. Intuitively, firms might be indifferent in multiple different financing decisions because of nonconvexities. The nonexistence resulting from nonconvexities can be avoided by randomizations over decisions. To mitigate the impact of the transitory income shock, I choose a relatively narrow standard deviation. Third, to reduce numerical errors, I employ interpolation on the pre-default equity value function S_{it}, avoiding interpolation near the kink in the post-default equity value function V_{it}, which is bounded below by zero. Although these strategies are generally

helpful, there are instances where they may fail.[4] To ensure credible model solutions, I experiment with a wide range of parameter values.

The benchmark parameter values are presented in Table 1. The time discount factor β is set to be 0.955 to match the average annual risk-free rate. This value is lower than those reported in previous literature due to the introduction of disaster risk. The high marginal utility in the disaster state reduces the risk-free rate, necessitating a lower time discount factor to obtain an average risk-free rate consistent with the data. The relative risk aversion, $\gamma = 7.5$, and the EIS, $\psi = 2$, are chosen to match the annual equity premium.

For the consumption dynamics, I match the key parameters to the consumption growth data from the National Income and Product Accounts Table 7.1 Chained (2012) dollars. I use data from 1947 to 2019 to eliminate the special effects of COVID-19. If including the COVID-19 period, the autocorrelation of consumption growth becomes -0.1, which is significantly different from the value of 0.3 in normal times as documented in the literature. Based on the empirical observations of consumption growth, I set the unconditional mean $\bar{g}$ to be 0.0036, the persistence or autocorrelation ρ_g to be 0.3, and the standard deviation σ_g to be 0.005. To ensure that the persistence and conditional volatility of consumption growth are close to those of productivity growth, following Bai et al. (2019), I set the relative volatility of aggregate productivity growth, $\phi = 1$.

For the parameters that govern the disaster dynamics, I set the persistence of the disaster state θ to be 0.914, which is the same as the value in Gourio (2012). The probability of entering the disaster state from normal states η is 0.028/4, implying an annual disaster probability of 2.8% and a quarterly disaster probability of 0.7%. The value is close to Gourio (2012) and Nakamura et al. (2013). To find the persistence of the recovery state ν, the size of the consumption crash in the disaster state λ_D, and the size of the recovery state λ_R, I experimented with different values to match the cumulative consumption growth pattern reported in Bai et al. (2019), Gourio (2012),

[4] In cases without convergence, the distance between the value function for this and the previous iteration tends to oscillate between some repeated values after reaching some level, for example, 10^{-2}.

and Nakamura et al. (2013). The procedure yielded $\nu = 0.88$, $\lambda_D = -0.07$, and $\lambda_R = 0.037$. Figure 2 reports the average and median of simulated cumulative consumption growth and its comparison with real data (bottom figure from Gourio (2012)). The maximum short-term negative shock is approsimately 20%, and the long-term decline is approximately 15%.

To calibrate the firm-side parameters, I set most of the values following what is reported in the literature. The curvature parameter, $\alpha = 0.65$, follows Hennessy and Whited (2007). The quarterly depreciation rate, δ, is 0.03 as in Kuehn and Schmid (2014) and Cooley et al. (1995). The tax rate on output, $\tau = 0.3$, is close to the values reported in Hennessy and Whited (2007). The coupon rate is set to 2%. Following Kuehn and Schmid (2014), the parameter that governs bond maturity, κ, is set to be 0.05 to reflect the five-year maturity. Similar to the values reported in Kuehn and Schmid (2014), I set the persistence of the firm's idiosyncratic volatility, ρ_z, to be 0.85, its volatility, σ_z, to be 0.12, and the recovery rate, $\xi = 0.8$, when default happens.

The remaining five parameters are calibrated to match a list of various moments of corporate policies, along with the average historical default rate. The asymmetric investment adjustment costs, $c^+ = 0.4$ and $c^- = 2.8$, are set to match the average and volatility of the investment-to-asset ratio. A relatively high divestment adjustment cost reduces both the level and volatility of the investment-to-asset ratio. As it is more costly to divest in bad times, this discourages firms from investing in normal times. The average long-run idiosyncratic productivity, $\bar{z} = -1.8$, is set to roughly match the default rates. The fixed cost of operation, $f = 0.003$, is further calibrated to more accurately match the moments. I start calibrating the parameters that govern financing frictions following Kuehn and Schmid (2014). The bond adjustment fixed cost ($\phi_0 = 0.03$) and proportional cost ($\phi_1 = 0.078$) are set to match the bond issuance moments, both the frequency and the average amount scaled by the capital level. The equity issuance costs, $\lambda_0 = 0.06$ and $\lambda_1 = 0.03$, are calibrated accordingly to match the equity issuance moments.

4.3 Aggregate and Firm-Level Moments

The benchmark parameter values of the representative consumer and firms are reported in Table 1. I independently simulate 5000 samples at a quarterly frequency, each with 5000 firms and 156 quarters (39 years) to match the length of the data. Among the 5000 samples, those without disasters are defined as "normal times." Because the COMPUSTAT data starts in 1984, which does not cover the disaster states in the US, the parameters are calibrated to match key moments in normal times. The key variables include the cross-sectional average and dispersion of the investment-to-asset ratio, the cross-sectional frequency, and the average amount of equity and bond financing, the cross-sectional average and dispersion of market leverage, and the cross-sectional average of default rates and credit spreads.

Table 2 reports firm-level moments. The most important moments are the default rates and credit spreads because they are the focus of this study. Bai (2021) reports an average annual default rate of 0.7%, and Kuehn and Schmid (2014) reports a cumulative five-year default rate of 2%. My model generates an annual default rate of 0.66% and a five-year default rate of 3.34%, which is overall consistent with the data.

The average credit spread between Baa-rated and Aaa-rated corporate bonds is reported to be 70-100 bps in Bai (2021) and 100 bps in Kuehn and Schmid (2014). My model generates an average credit spread of 92.79 bps and a 90% confidence interval range between 19.52 and 161.90, which covers the empirical observations.

The market leverage in the data is approximately 24.3%, while Bai et al. (2019) reports an average market leverage of 30% using international data. My model produces an average market leverage of 31.1%. This result is higher than the empirical observation obtained using Compustat data, but it is close to the number derived using international data. The 90% confidence interval is between 6.3% and 54.6%, covering both numbers.

The model can successfully match the low default rate and market leverage but high credit

spread for two reasons. First, the underlying disaster risk generates high risk premiums and potential default risk, generating high credit spreads. As long as a disaster does not materialize, the realized default rates stay low. Second, long-term bonds serve as hedge instruments, generating relatively high firm equity value. Although long-term bonds incentivize firms to use more bonds, it does not generate high market leverage.

My simulation results overall match the investment and financing moments. The model-simulated average investment-to-asset ratio is 0.029, and its volatility is 0.095, which are close to the empirical moments of 0.033 and 0.111, respectively. The equity issuance and bond issuance moments are also overall close to the data, while the average amounts are slightly off. The average amount of new equity is lower in the model simulation than in the data, while the new bond issuance amount is higher. This discrepancy is due to the attractiveness of long-term bonds under the threat of disaster risk. The average frequencies of new equity and bond issuance are 0.069 and 0.274 in the data, and the simulation results are 0.181 and 0.268, respectively. Both results fall within the 90% confidence interval. The cyclicality moments are from Bai (2021), including the correlation between consumption growth and average default rate, credit spread, and market leverage. My model generates results that are quantitatively consistent with the data.

The real and financial friction parameters jointly affect investment and financing strategies. Lower investment adjustment costs lead to a lower average investment ratio because firms can easily divest their capital, which also generates higher volatility in the investment ratio. It also leads to more aggressive financing strategies because it is easier for firms to liquidate capital. Reduced financial frictions have similar effects, as it becomes easier to issue new equity and bonds.

Table 3 reports the moments of equity premium and risk-free rate in samples with realized disasters. The reason is that the equity market data starts from 1872, including World War I and the Great Depression, the two identified disasters in the United States. My model generates a 3.73% risk-free rate and 9.09% aggregate equity premium, which are close to the empirical observations in

the data. The model generates lower average volatilities of the risk-free rate and aggregate equity return than in the data. However, my 90% confidence interval covers the data moments, showing a reasonable performance of the model. The reason for the low equity volatility is that I do not model strongly time-varying disaster probabilities as in Nakamura et al. (2013). As such, my model has limited variation in the equity premium over time, resulting in low equity volatility. This limitation is well acknowledged in the disaster risk literature (Bai et al., 2019). This modeling decision is for parsimony only. Introducing time-varying disaster probabilities is conceptually straightforward (albeit computationally challenging).

To more explicitly depict the economic mechanisms, I plot the policy functions of the two endogenous state variables, capital (K) and bonds outstanding (B) in Figure 3. When plotting the policy function with respect to one state variable, I fix the other state variables at the average level, except for consumption growth. The policy functions are plotted with both average consumption growth in normal times and disaster states to highlight their differences.

Figure 3 Panel (a) plots the investment to capital decision with respect to K and B. The blue line represents the consumption growth shock being zero, and the red line is when the economy is in a disaster state. Firms invest more when the capital level is low and divest to obtain liquidity when a disaster hits. In contrast, firms with sufficient capital levels divest less because they have enough capital to generate profits and liquidity to survive the disaster without divestment. Similarly, they divest more under greater bonds outstanding to repay the predetermined principal and coupon.

Figure 3 Panel (b) plots the bond issue to the total bonds outstanding decision. Firms issue more bonds when the capital level is low because they cannot rely on output to maintain operations and repay liabilities. In a disaster state, if the firm does not default, the firm issues more bonds to mitigate the large negative productivity and output shock. Firms issue bonds when they have more bonds outstanding, showing a tendency to use new bonds to repay old bonds.

There is heterogeneity across firms' policy functions when they have different amounts of capi-

tal and bonds outstanding. Figure 4 plots the investment decisions for small (lowest K) and large (highest K) firms and firms with more (highest B) or less (lowest B) bonds outstanding. Small firms default when their bonds outstanding are too high. If they do not default, they invest substantially to accumulate more capital. In comparison, in normal times, large firms invest more when their bonds outstanding are not too high and invest less if they have high liability outstanding. They divest when a disaster hits regardless of their bond level.

Firms with different levels of bonds outstanding have different investment decisions. The investment decision with respect to the capital level generally shows a convex pattern. Firms with less capital invest to accumulate more capital, and firms with substantial capital invest less because of the decreasing return to scale. Firms with moderate amounts of capital divest to mitigate the negative productivity shock because they lack sufficient capital to produce and meet operation and liability requirements. Firms with fewer bonds outstanding will generally invest and divest when a disaster hits. However, firms with more bonds outstanding but not high enough capital will divest substantially to repay the liabilities because of the sudden decrease in output.

Figure 5 plots the bond issuance decisions. Small firms, if they do not default, are bond issuers and rely on new bonds to repay old bonds. They are more likely to default and issue fewer bonds in a disaster state. Large firms, on the other hand, are savers. They rely more on capital and output to operate. Firms with fewer bonds outstanding do not default. In the disaster state, they issue more bonds when the capital level is low but repay the bonds when the capital level is high. In comparison, firms with more bonds outstanding default when the capital level is low. If they do not default, they always issue bonds, especially when the capital level is low.

In this section, I show that the model performs reasonably well in matching the key aggregate and firm-level moments. I also present the underlying policy functions that generate the results. The following sections use this calibrated model to explore other implications of the model.

4.4 Disaster Risk Implications

As a solution to the "credit spread puzzle," the benchmark results successfully match the default rate, credit spread, and equity premium in the data. To further explore the underlying economic mechanisms, in this section, I discuss the effects of disaster risk on credit spreads in normal times. I focus on three dimensions: disaster probability (η), the severity of the disaster (λ_D), and the persistence of the disaster (θ). I separately explore the model under higher and lower η, λ_D, and θ while keeping all other factors fixed.

As a comparative statistic, I explore parameter values that differ from but are close to the benchmark values. I do not completely shut down disaster risk because the results would be fundamentally different. The aggregate and corporate policy moments would deviate significantly from the benchmark moments, making it difficult to compare the default rate and credit spread results. Therefore, I focus on exploring the changes when the disaster risk becomes more or less severe while keeping it present in the model.

Table 4 reports the model simulation results in different disaster risk scenarios. Overall, increased disaster risk (a higher probability, greater severity, or higher persistence) leads to higher credit spreads in normal times. Two main economic mechanisms drive these results. First, greater disaster risk is associated with a higher risk premium and expected default rate. Second, firms adopt different bond financing strategies in response to different levels of disaster risk. When the disaster risk is more severe, firms face difficulty accumulating capital as investors become more hesitant to invest in risky assets. Firms have to issue new bonds to maintain liquidity and roll over existing bonds. Moreover, long-term bonds serve as a hedging instrument for firms due to lower promised principal repayment and potential recovery states. With a given amount of bonds, a lower capital level leads to a lower equity value and, therefore, higher leverage. This combination of factors causes higher credit spreads and higher default rates when the disaster risk is more severe.

Using the benchmark parameter values, the model generates an average annual default rate of

0.66%, a five-year default rate of 3.34%, an average credit spread of 92.79 bps, and an average market leverage of 31.1%. First, I study two disaster probability scenarios. Given a lower disaster probability. With lower disaster probability, the average annual default rate decreases to 0.24%, and the average credit spread decreases to 30.36 bps, along with an average leverage of 12.3%. In contrast, with a higher disaster probability, the model generates an average annual default rate of 2.41%, an average credit spread of 159.68 bps, and an average market leverage of 42.8%.

Then, I study two scenarios of the consumption growth crash. If I decrease λ_D, which means a model with a more severe consumption growth crash when a disaster is realized, the model generates an annual default rate of 1.36%, an average credit spread of 254.03 bps, and an average market leverage of 0.86%. In contrast, a higher λ_D, or a less severe consumption growth crash, leads to an annual default rate of 0.55%, an average credit spread of 33.96 bps, and an average market leverage of 9.8%.

Last, I study two scenarios regarding the persistence of the disaster. If the disaster risk is less persistent, with lower θ, the annual default rate decreases to 0.45%, the average credit spread decreases to 21.18 bps, and the market leverage decreases to 19.5%. In contrast, with more persistent disaster risk, or higher θ, the model generates an annual default rate of 2.12%, an average credit spread of 138.09 bps, and an average market leverage of 37.2%.

4.5 Disaster States

In this section, I report the model predictions during disaster states. Tables 5 and 6 present the key moments in the simulation samples where disasters occur. Compared to normal times, more firms default, and credit spreads increase significantly. The annual default rate is more than ten times higher than in normal times, with a 90% confidence interval from 1.13% to 27.10%. The average credit spread more than triples its average in normal times, with a 90% confidence interval from 111.25 bps to 652.73 bps This is consistent with the observations around the Great Depression as plotted in Figure **??**. During the Great Depression, the annual default rate peaked at 8% and the credit spread was as high as 6%.

Realized disasters vastly affect corporate policies. Surviving firms reduce investments, or even divest, to compensate for the substantial negative cash flow shock. The average investment ratio becomes slightly negative, -0.005, and the cross-sectional volatility of investment increases to 0.14. Bond financing becomes more challenging than in normal times due to financing frictions, low expected recovery rates, and high expected default rates. The average frequency and amount of new bond issuance both decline to 0.22. Equity financing is almost frozen, with the frequency of new equity issuance dropping to 0.021, and the average amount dropping to 0.008.

To provide a more explicit demonstration of the economic mechanisms, I simulate a long sample of 100,000 quarters after dropping the burn-in period of 1000 quarters and plot key variables with respect to consumption growth. The key variables are depicted in heatmaps to show the density of observations. As both disaster and recovery states are discrete in my model, the variables are plotted vertically. In normal times, I simulate the economic states in a more continuous way. In each figure, I report the correlation between variables and consumption growth to illustrate how they vary with different macroeconomic conditions.

Figure 6 displays the dynamics of default rates and credit spreads. Both have a strong negative correlation with consumption growth. In the disaster state, the average default rate and credit spread are significantly higher. However, the range is wide because of different states before the economy enters the disaster state. If the economy were performing well and firms were financially stronger, there would be less default, and the credit spread would be lower when a disaster is realized.

Figure 7 shows the dynamics of investment. The investment-to-capital ratio is positively correlated with consumption growth, while its dispersion has a strong negative correlation. Firms liquidate capital when there is a large negative productivity and cash flow shock to maintain operations and repay bond principal. The divestment incentives vary substantially across firms, depending on their financial conditions, which leads to large cross-sectional dispersion.

Figure 8 displays the dynamics of bond and equity financing. Equity financing is positively correlated with consumption growth. In the disaster state, the new equity issuance frequency and amount are very close to zero. The new bond issuance frequency is also close to zero in the disaster state, but the amount has a negative correlation with consumption growth. However, most observations are clustered around zero. The negative relationship is driven by the disaster following good economic conditions. Surviving firms are financially strong and can exploit long-term bonds when a disaster hits. Therefore, the amount of new bond issuance in the disaster state has a wide range, which generates a negative correlation.

Figure 9 demonstrates the dynamics of market leverage and recovery rate. Market leverage is negatively correlated with consumption growth, while the recovery rate has a positive correlation. The high market leverage is driven by plummeting of firm values and persists in the recovery state. The recovery rate has a similar but opposite pattern, driven by the large negative productivity and cash flow shock.

4.6 Decompose the Credit Spread

The results of previous sections show that in both normal times and realized disaster states, credit spreads change along with endogenous leverage choices. Moreover, as shown in the literature, credit spreads are determined by the expected default probability and loss and the risk premium. In this section, I follow Kuehn and Schmid (2014) to decompose the credit spreads into three channels (default risk, risk premium, and leverage) and study their relative importance. Specifically, I restructure Equations (7) and (8) and define the "expected loss in default" (hereafter expected loss) as

$$\mathrm{E}_t[\mathbb{1}^D_{it+1}] - \frac{\mathrm{E}_t[R_{it+1}\mathbb{1}^D_{it+1}]}{R_f} \tag{29}$$

which is the difference between the expected default probability and the expected recovery rate if default occurs. To understand the intuition, consider a special case where the risk-free rate R_f is 1, and there is no correlation between the recovery rate, R_{it+1}, and default, $\mathbb{1}^D_{it+1}$. The expected

loss will be simplified to $\mathrm{E}_t[\mathbb{1}^D_{it+1}](1-\mathrm{E}_t[R_{it+1}]) \equiv$ Prob(Default)×LGD (loss given default). The higher (lower) the default rate (recovery rate) is, the higher the expected loss is. I use this term to represent the degree of default risk.

The "risk premium" is defined as

$$\mathrm{cov}_t\left(\frac{M_{t+1}}{\mathrm{E}_t[M_{t+1}]}, \mathbb{1}^D_{it+1}\right) - \mathrm{cov}_t\left(M_{t+1}, R_{it+1}\mathbb{1}^D_{it+1}\right) \tag{30}$$

which captures the covariance between the marginal utility and default. In bad times, marginal utility and default probability are high, while the recovery rate is low. Therefore, the first term is positive and the second term is negative, leading to a positive "risk premium".

The definitions above are based on a model with one-period bonds because there is no closed-form solution for long-term bonds. Although not perfect, they are proxies for the default risk and risk premium, which can be used to shed light on how much the credit spreads are explained by the three channels. Furthermore, the default risk and risk premium are firm-specific, giving explanations for firm-level credit spreads.

To decompose the credit spreads, I simulate a panel of firms and regress credit spreads on market leverage, the expected loss, and the risk premium. I use different combinations of the explanatory variables and explore their incremental contribution to R^2. The coefficients and R^2 are averaged across 5000 independent simulations. Following each coefficient estimate, I also report the 5% and 95% percentiles of the estimate across all simulations to show the confidence interval. I report results from pooled panel regressions with and without time-fixed effects.

Table 7 reports the results in normal times and times with realized disasters. Intuitively, all the coefficients on explanatory variables are positive. In normal times, the risk premium has the strongest explanatory power, with an R^2 of 0.55. In comparison, market leverage and the expected loss alone have R^2 of 0.29 and 0.39, respectively. The combination of the risk premium and the expected loss explains approximately 90% of the variation in the credit spread. Adding time fixed

effects only marginally increases the R^2. The results are almost unchanged.

In times of realized disasters, the expected loss has the strongest explanatory power. It alone has an R^2 of 0.86, showing that default risk plays the largest role in explaining credit spreads. Adding time fixed effects increases the explanatory power of all factors, especially market leverage and the risk premium. However, the results are qualitatively unchanged: default risk still plays the largest role. Intuitively, in realized disaster periods, the default probability is expected to be substantially higher because of the persistent disaster state. Moreover, the recovery rate is considerably lower if a default occurs because of the plummet in productivity. In contrast, market leverage has limited explanatory power. When a disaster is realized, all firms suffer from the productivity shock. The large negative shock drives firm values directly to negative levels, regardless of their leverage.

The comparative statics in Table 4 show that different degrees of disaster risk have direct effects on both the market leverage and credit spread. The first impression of the result is that market leverage drives the change in credit spreads. However, this is only part of the story. Table 7 shows that the risk premium and the expected loss jointly explain a large portion of the credit spreads.

Thus, is the market leverage redundant? The answer is no, because it is still strongly significant after controlling for the risk premium and expected loss. The exogenous shocks to the risk premium and the expected loss are transferred into the endogenous choice of leverage. The endogenous leverage decision includes adjustments of next-period capital and bond levels, which determine the capital structure in the next period. In other words, although the contemporaneous credit spread is more driven by the exogenous shock, the next-period credit spread depends more on the choice of capital structure. Specifically, the regressions on next-period credit spreads show that market leverage plays the largest role in normal times. In comparison, when disasters are realized, default risk still plays the largest role, and the second explanatory variable now becomes market leverage.

It is also important to understand what the endogenous capital structure depends on. First, I regress market leverage on the risk premium and expected loss. The first three columns in Table 9

report the regression results. Intuitively, the coefficients are both positive and significant. Higher risk premiums and expected losses are transformed into higher market leverage. The regression results show that the risk premium has more explanatory power for market leverage, both in normal times and disaster states. However, the combination of both only explains less than 20% of the variation in market leverage, showing that market leverage cannot be subsumed.

To show how market leverage is affected by other firm fundamentals, I report regressions using other related variables. Table 9 shows that higher risk premiums and expected losses are transformed into lower capital levels and firm equity values, increasing market leverage. The risk premium has stronger effects than the expected loss. They also lower firm bond issuance, leading to a more conservative financing strategy.

In summary, the credit spread can be decomposed into three channels: the risk premium, default risk, and market leverage. Consistent with previous bond pricing models, the first two channels have strong explanatory power for contemporaneous credit spreads. They also increase market leverage by lowering the firm capital level and equity value. The heightened market leverage will be transmitted into next-period credit spreads. Endogenous dynamics of market leverage contribute to the credit spreads in normal times but have limited impacts in disaster states.

4.7 Cross-Sectional Analysis

The decomposition of credit spreads highlights interesting results at the firm level. Firm characteristics are important in explaining credit spreads in both normal times and disaster states. In this section, I study more cross-sectional implications. First, I verify whether my model generates cross-sectional implications that are generally consistent with previous literature (Gomes and Schmid, 2021). Second, I explore how different firms react to the realization of disasters.

Table 10 reports results from regressing credit spreads and market leverage on other firm characteristics. The data column is from Gomes and Schmid (2021). Overall, the credit spread column is consistent with the data, except for the size. Higher market leverage increases credit spreads,

while higher profitability decreases credit spreads. In my model, larger firms have more capital and output. Ceteris paribus, larger firms have lower credit spreads. However, the market leverage column is counterintuitive. The size and profitability coefficients are both the opposite of empirical observations. The reason is that larger firms are more productive and have higher equity value, which decreases market leverage. When firms are profitable, they have more capacity to hold more bonds, which increases market leverage. This is the opposite of the empirical finding that firms, in good times, repay bonds to increase borrowing capacity. This is because my model does not include a strong financial flexibility channel as in, for example, DeAngelo et al. (2011).

Other than the relationship between contemporaneous characteristics, firms respond differently when a disaster hits. For example, immediately after the emergence of COVID-19, financially more flexible firms lost less value in their stock prices and suffered less during the pandemic period (Fahlenbrach et al., 2021; Stulz, 2023). Figure 10 plots the impulse response functions of average firm equity value and credit spread. I separately plot firms with more or less financial flexibility, which is defined as market leverage [5]. The results are qualitatively the same as the empirical findings. More financially flexible firms have a smaller decline in equity value and a jump in credit spreads. In a recovery state, the lost equity value is partially recovered, and credit spreads decrease to the levels in the pre-disaster period. Without a recovery state, the lost equity value and increased credit spreads are not restored.

Table 11 reports regression results of the collapse period equity return and credit spread on other firm characteristics. The collapse period is defined as the first quarter when a disaster is realized. Intuitively, more profitable firms lose less equity value, and the credit spreads increase less. Conditional on market leverage, larger firms lose less equity value. However, they face high divestment costs to liquidate capital, which is transmitted into higher next-period credit spreads.

[5] Fahlenbrach et al. (2021) uses cash and book leverage to define financial flexibility. I use market leverage here for two reasons. First, I do not have cash in my model, and market leverage is well-calibrated to the data moments. Second, market leverage sheds light on the channel of equity issuance as a financing instrument.

4.8 Effects of Other Economic Forces

In this section, I use comparative statics exercises to study how various economic forces affect credit spreads. I first explore firm-side parameters, and then analyze consumer-side parameters. On the firm side, I focus on real and financial frictions during normal times. I analyze the model by re-solving it with different parameter values that govern investment adjustment costs and financing costs while keeping all other factors constant. Contrary to the findings in Kuehn and Schmid (2014), I find opposite effects of real and financial frictions. There are two opposite economic mechanisms behind the results. When there are less frictions, firms lever up because it is easier to roll over existing bonds or divest to repay principal. Moreover, as argued in Kuehn and Schmid (2014), less frictions make bonds less risky. In their paper, they find that real and financial frictions increase credit spreads because the latter mechanism dominates the former. However, in my model, the first mechanism dominates. With more friction, both the default rate and credit spread decrease. When it is more costly to roll over bonds or issue equity, the hedging motive becomes less attractive. As a result, firms choose more conservative bond financing strategies, and market leverage drops. Conversely, with less friction, firms choose more aggressive financing strategies because it becomes easier to roll over bonds.

Additionally, the effects of the recovery rate (ξ) and fixed operating cost (f) are more intuitive. A lower recovery rate or a higher fixed operation cost leads to an increase in the default rate and credit spread. These findings shed light on how real and financial frictions, and other economic forces, play a crucial role in determining credit spreads in the model, providing a better understanding of the economic mechanisms during normal times.

Table 12 reports the detailed comparative statics during normal times. In Panel A, we observe the effects of more friction, including higher investment adjustment costs and financing costs, lower recovery rates, and higher fixed operating costs. Higher investment adjustment costs cause lower leverage, leading to less default and a lower credit spread. The credit spread decreases from 92.79

bps to less than 40 bps. Similar effects are observed with higher financing costs. Even small changes in fixed equity and bond financing costs have strong effects, reducing the credit spread to less than 20 bps. Lower proportional financing costs have similar but more moderate effects. Lower proportional equity issuance costs lower the credit spread to 70.11 bps, and lower proportional bond issuance costs lower it to 35.54 bps. A lower recovery rate increases the credit spread to 348.15 bps, while a higher fixed operating cost increases it to 150.82 bps.

Panel B of Table 12 presents the effects of less friction, which are generally the opposite of those in Panel A. Lower investment adjustment costs encourage more investments during normal times. However, firms with more capital become riskier due to the asymmetric investment adjustment costs. It becomes costly to divest with a negative shock and firms need to liquidate capital. Higher investment costs increase the credit spread to 93.51 bps, and higher divestment costs increase it to 164.80 bps. Lower costs of equity and bond financing increase default rates and the credit spread. As it becomes cheaper to finance, firms opt for more aggressive corporate policies. Higher fixed (proportional) equity issuance costs increase the credit spread to 256.32 bps (161.60 bps). Lower fixed (proportional) bond issuance costs increase the credit spread to 222.92 bps (225.47 bps). Conversely, a higher recovery rate lowers the credit spread to 23.19 bps, and a lower fixed operating cost lowers it to 74.64 bps.

The effects of real and financial frictions in the disaster state are more limited because default risk dominates other economic forces. Although limited, the pattern of the default rate, credit spread, and market leverage is similar to the pattern in normal times. With more friction, the average credit spread is approximately 200 bps, slightly lower than the benchmark, and the annual default rate is approximately 10%, which is close to the benchmark result. Table 13 reports the detailed results and shows that the negative aggregate productivity shock has a more direct and critical impact, while frictions only have marginal effects when a disaster hits.

Now, I explore the effects of the stochastic discount factor. Table 14 reports the comparative

statics of key parameters that govern the dynamics of the stochastic discount factor. Panel A reports the results with higher parameter values, with everything else held fixed. Panel B reports results with lower parameter values. I focus on the impacts on market leverage, the default rate, and credit spreads.

Overall, more patient (higher β), more risk averse (higher γ), and more volatile aggregate productivity (higher ϕ) lead to higher credit spreads. However, the economic mechanisms are slightly different. More patience leads to lower default rates because it directly increases firm value in all states and decreases the probability of default. Firms choose more aggressive financing decisions, which generates higher market leverage and then causes credit spreads to increase. In contrast, more risk aversion and a more volatile aggregate productivity shock decrease firm value and increase default probability, which also generates higher market leverage and credit spreads.

Higher EIS (ψ), persistence (ρ_g), and volatility (σ_g) of consumption growth generate slightly higher default rates, lower market leverages, and lower credit spreads. When ψ is high, the representative consumer is more willing to accept fluctuations, which leads to more diverse firm values and generates higher default rates. In response, firms choose more conservative financing policies. The capital structure decision dominates the default risk in normal times and generates lower credit spreads. When consumption growth (and aggregate productivity growth) is more persistent, firms' output is also more persistent. Firms have lower market leverage, which generates lower credit spreads. When consumption growth (and aggregate productivity growth) volatility is high, firms are more likely to have negative continuation values, which generates high default rates. In response, firms adopt more conservative financing policies, which leads to lower credit spreads.

In summary, both firm-side frictions and consumer-side preferences affect credit spreads. More real and financial friction leads to lower credit spreads. Overall, the effects of market leverage dominate the effects of default rates. The changes in credit spreads align with the direction of the changes in market leverage, which highlights the impacts of endogenous corporate policy choices

on credit spreads.

5 Conclusion

Disaster risk directly impacts corporate credit spreads by affecting the default risk and risk premium. Additionally, it critically influences a firm's capital structure due to its effects on corporate policies. To explore how disaster risk affects credit spreads, I develop a dynamic structural model with long-term bonds that rationalizes corporate investment, financing, and default decisions. The model simulations are divided into economies with and without realized disasters.

In normal times with an underlying disaster risk, higher disaster risk leads to a higher default risk and a more nonlinear pricing kernel. Moreover, firms have higher leverage because they face difficulty in accumulating capital and experience lower equity value. Without enough capital and output, firms are forced to issue new bonds to roll over existing bonds and maintain bond levels. Furthermore, long-term bonds offer the opportunity to exploit subsequent recovery states and have less pressure on principal repayment. These factors collectively result in higher market leverage and credit spreads on average. When disaster risk is realized, the model predicts a significantly higher credit spread, default rate, and market leverage. Compared to normal times when the risk premium plays the largest role in explaining credit spreads, default risk has the most explanatory power in disaster states. I also explore the effects of other economic forces, with a focus on real and financial frictions. The results differ from previous findings when there is disaster risk, mainly due to their varying impacts on firm leverage.

Solving a dynamic model with a long-term bond is computationally challenging. To keep the setting tractable, I abstract from many interesting and realistic aspects, such as corporate cash holding and time-varying disaster risk. It would be useful to study more interactions between disaster risk and other aspects of corporate finance, and the implications for asset prices, in future research. Here, I discuss three potential extensions.

First, the model could be extended to study liquidity management and cash holdings. During disaster states, cash has an important role in mitigating the negative productivity shock. Cash is fundamentally different from negative long-term bonds because it has short maturity. Moreover, with the recovery state, long-term bonds serve as a hedging instrument against disaster risk, while cash provides contemporaneous liquidity. A model that incorporates both cash and bond holding policies will be important in understanding the different values of cash and bonds in disaster states.

Second, the model can be extended to incorporate more realistic time-varying disaster risk. Rather than forming a constant objective belief in disaster risk, investors update their belief on the probability of a disaster and the losses when a disaster hits. For example, before a financial crisis hits, investors closely follow news and macroeconomic metrics to form expectations with updated information. The updates in beliefs should affect corporate bond prices.

Third, the model can be extended to understand credit expansion and how the endogenous expansion leads to a disaster. As shown in the financial crisis literature, a financial crisis is best predicted by the interaction of credit expansion and asset price growth. Long-term bonds serve as a hedging instrument against disasters, motivating firms to lever up. With overall higher leverage, the disaster risk becomes more severe. Such amplification effects should have important implications for corporate bond prices in the cross-section.

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Table 1: Benchmark Parameter Values

This table lists the benchmark parameter values used to solve and simulate the model at a quarterly frequency. I break all the parameters into two groups. The top group includes parameters from the utility function of the representative consumer. The bottom group includes parameters from the firm side. For each parameter, I list its notation, value, and explanation.

Parameter	Value	Meaning
	Consumer	
β	0.955	Subjective discount factor
γ	7.5	Relative risk aversion
ψ	2	Elasticity of intertemporal substitution
$\bar{g}$	0.0036	Average consumption growth
ρ_g	0.3	Persistence of consumption growth
σ_g	0.005	Volatility of consumption growth
η	0.028/4	Probability of entering a disaster state
θ	0.914	Persistence of disaster state
ν	0.88	Persistence of recovery state
λ_D	-0.07	Severity of disaster state
λ_R	0.037	Size of recovery state
	Firm	
α	0.65	Concavity of production function
δ	0.03	Depreciation rate
ϕ	1	Relative volatility of aggregate productivity
τ	0.3	Tax rate
c	0.02/4	Coupon rate
κ	0.05	Bond maturity
ρ_z	0.85	Persistence of idiosyncratic productivity
σ_z	0.12	Volatility of idiosyncratic productivity
ξ	0.8	Recovery rate of bond
c^+	2.6	Investment adjustment cost
c^-	0.4	Divestment adjustment cost
$\bar{z}$	-1.8	Average idiosyncratic productivity
λ_0	0.06	Fixed cost of equity issuance
λ_1	0.03	Proportional cost of equity issuance
ϕ_0	0.03	Fixed cost of bond issuance
ϕ_1	0.078	Proportional cost of bond issuance
f	0.003	Fixed cost of operation

Table 2: Firm-Level Moments and Cyclicality

This table reports a set of key firm-level moments generated under the benchmark parameters reported in Table 1 in normal times, being consistent with the data. The data moments for investment, equity issue, bond issue, and market leverage moments are calculated from CRSP-Compustat Merged. The 1-year default rate data moment is from Bai (2021). The 5-year default rate and credit spread data moments are from Kuehn and Schmid (2014). The simulation moments are calculated across 5000 independent simulations, which do not have realized disasters. Each simulation has 156 quarters (39 years).

	Data	Mean	5%	25%	50%	75%	95%
Average Investment-to-Asset	0.033	0.029	0.026	0.028	0.029	0.031	0.033
Volatility Investment-to-Asset	0.111	0.095	0.051	0.078	0.099	0.115	0.128
Frequency of New Equity Issue	0.069	0.181	0.037	0.109	0.185	0.257	0.316
Average New Equity to Asset	0.176	0.098	0.096	0.097	0.098	0.099	0.100
Frequency of New Bond Issue	0.274	0.268	0.054	0.159	0.274	0.379	0.471
Average New Bond to Asset	0.071	0.368	0.363	0.366	0.368	0.369	0.373
Average Market Leverage	0.243	0.311	0.063	0.186	0.317	0.439	0.546
1 Year Default Rate	0.7 (%)	0.656	0.131	0.394	0.663	0.92	1.166
5 Years Default Rate	2 (%)	3.341	0.683	2.023	3.383	4.675	5.918
Credit Spread	100 (b.p.)	92.792	19.524	55.809	94.273	130.941	161.897
Cyclicality							
Default Rate	-0.33	-0.183	-0.314	-0.236	-0.186	-0.131	-0.049
Credit Spread	-0.36	-0.417	-0.536	-0.47	-0.422	-0.367	-0.285
Market Leverage	-0.11	-0.014	-0.186	-0.085	-0.018	0.055	0.163

Table 3: Aggregate Moments

This table reports a set of key aggregate and cyclical moments generated under the benchmark parameters reported in Table 1 in samples with realized disasters, being consistent with the data. The cyclicality is the correlation between the variable and the consumption growth. All numbers are annualized. The data moments for the average and volatility of the market excess return are from the CRSP value-weighted market return. The data moments for the average and volatility of the risk-free rate are from Kenneth French's website. The cyclicality moments are from Bai (2021). The simulation moments are calculated across 5000 independent simulations, which do not have realized disasters. Each simulation has 156 quarters (39 years).

	Data	Mean	5%	25%	50%	75%	95%
Annualized Equity Premium	7.063	9.092	6.566	7.947	8.836	9.768	12.471
Volatility of Equity Premium	19.388	8.925	3.002	5.137	7.126	10.645	19.681
Annualized Risk-Free Rate	3.614	3.733	2.808	3.031	3.458	4.154	5.562
Volatility of Risk-free Rate	2.567	1.583	0.553	1.095	1.536	2.109	2.812

Table 4: Disaster Risk Implications in Normal Times

This table reports the impacts of disaster risk in normal times, focusing on three parameters that govern the disaster risk: disaster probability η, size of consumption crash in disaster states λ_D, and disaster persistence θ. For each parameter, I study both more and less disaster risk. Panel A reports the four key moments under the benchmark parameters: 1-year cumulative default rate, 5-year cumulative default rate, average credit spread, and average market leverage. Panel B and Panel C report the comparative statics with lower and higher disaster probability. Panel D and Panel E report comparative statics with more and less severe consumption crashes in disaster states. Panel F and Panel G report comparative statics with more and less persistent disasters. The default rates are reported in percentages. The credit spread is reported in basis points. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters.

	Data	Mean	5%	25%	50%	75%	95%
Panel A: Benchmark ($\eta = 0.007$, $\lambda = -0.07$,$\theta = 0.914$)							
1 Year Default Rate	0.7 (%)	0.656	0.131	0.394	0.663	0.92	1.166
5 Years Default Rate	2 (%)	3.341	0.683	2.023	3.383	4.675	5.918
Credit Spread	100 (b.p.)	92.792	19.52	55.81	94.273	130.94	161.897
Market Leverage	0.243	0.311	0.063	0.186	0.317	0.439	0.546
Panel B: Low $\eta = 0.003$ (Lower Disaster Probability)							
1 Year Default Rate	0.7 (%)	0.239	0.013	0.017	0.167	0.408	0.728
5 Years Default Rate	2 (%)	1.231	0.066	0.091	0.865	2.092	3.721
Credit Spread	100 (b.p.)	30.36	2.651	3.089	21.419	51.006	90.46
Market Leverage	0.243	0.123	0.007	0.009	0.086	0.208	0.375
Panel C: High $\eta = 0.009$ (Higher Disaster Probability)							
1 Year Default Rate	0.7 (%)	2.406	1.482	2.022	2.449	2.821	3.201
5 Years Default Rate	2 (%)	11.639	7.215	9.843	11.887	13.543	15.315
Credit Spread	100 (b.p.)	159.68	82.06	126.8	162.5	195.14	223.54
Market Leverage	0.243	0.428	0.214	0.336	0.436	0.525	0.615
Panel D: Low $\lambda_D = -0.071$ (More Severe Disaster)							
1 Year Default Rate	0.7 (%)	1.363	1.109	1.286	1.38	1.463	1.564
5 Years Default Rate	2 (%)	6.627	5.37	6.222	6.7	7.109	7.673
Credit Spread	100 (b.p.)	254.03	205.5	252.7	260.99	266.88	273.06
Market Leverage	0.243	0.855	0.69	0.856	0.881	0.899	0.912
Panel E: High $\lambda_D = -0.069$ (Less Severe Disaster)							
1 Year Default Rate	0.7 (%)	0.552	0.012	0.106	0.399	0.909	1.541
5 Years Default Rate	2 (%)	2.892	0.065	0.567	2.13	4.774	7.971
Credit Spread	100 (b.p.)	33.96	1.681	7.313	24.854	55.259	92.033
Market Leverage	0.243	0.098	0.002	0.019	0.071	0.161	0.27
Panel F: Low $\theta = 0.85$ (Less Persist Disaster)							
1 Year Default Rate	0.7 (%)	0.445	0.13	0.37	0.633	0.872	1.095
5 Years Default Rate	2 (%)	2.297	0.672	1.888	3.222	4.46	5.555
Credit Spread	100 (b.p.)	21.183	20.37	55.08	93.311	128.82	159.436
Market Leverage	0.243	0.195	0.068	0.189	0.322	0.446	0.555
Panel G: High $\theta = 0.924$ (More Persist Disaster)							
1 Year Default Rate	0.7 (%)	2.121	1.246	1.681	2.119	2.57	2.978
5 Years Default Rate	2 (%)	10.301	6.088	8.232	10.293	12.446	14.365
Credit Spread	100 (b.p.)	138.09	64.18	101.7	138.85	175.92	208.795
Market Leverage	0.243	0.372	0.164	0.268	0.373	0.48	0.573

Table 5: Credit Spread and Default Rate in Disaster States

This table reports the default rates and credit spread in simulations with realized disasters. The first column lists the simulation moments in normal times for comparison. The default rates are reported in percentages. The credit spread is reported in basis points. The moments are calculated across 5000 independent simulations which include realized disasters.

	Normal Times	Disaster States					
	Mean	Mean	5%	25%	50%	75%	95%
1 Year Default Rate	0.656	10.445	1.131	4.125	8.117	14.602	27.104
5 Years Default Rate	3.341	25.207	3.9	15.548	20.655	32.977	55.2
Credit Spread	92.792	315.148	111.25	189.846	272.377	402.774	652.731

Table 6: Corporate Policies in Disaster States

This table reports the corporate policies in simulations with realized disasters. The corporate policies are the same as in Table 2. The first column lists the simulation moments in normal times for comparison. The moments are calculated across 5000 independent simulations which include realized disasters.

	Normal Times	Disaster States					
	Mean	Mean	5%	25%	50%	75%	95%
Average Investment-to-Asset	0.029	-0.005	-0.066	-0.02	0.003	0.018	0.027
Volatility Investment-to-Asset	0.095	0.14	0.092	0.124	0.14	0.157	0.187
Frequency of New Equity Issue	0.181	0.021	0	0	0	0	0.251
Average New Equity to Asset	0.098	0.008	0	0	0	0	0.098
Frequency of New Bond Issue	0.268	0.224	0	0	0.193	0.456	0.559
Average New Bond to Asset	0.368	0.218	0	0	0.375	0.408	0.447
Average Market Leverage	0.311	0.563	0.281	0.476	0.586	0.67	0.769

Table 7: Decompose the Credit Spread

This table reports the decomposition of the cross-sectional credit spread, by regressing credit spreads on market leverage (ML), the risk premium (RP), and the expected loss in default (Exp. Loss) using simulated data. Confidence intervals follow the coefficients. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The confidence intervals are calculated across simulations. The risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times														
	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)	(5)	(5)	(6)	(6)	(7)	(7)
Constant	0	0	0.004	0.004	0.008	0.009	0	0	0	0	0.003	0.004	0	0
5%	-0.001	-0.001	0.001	0.001	0.002	0.003	0	0	-0.001	-0.001	0.001	0.001	0	0
95%	0	0	0.007	0.007	0.015	0.015	0	0	0	0	0.006	0.006	0	0
ML	0.031	0.031					0.015	0.016	0.028	0.028			0.013	0.013
5%	0.03	0.03					0.015	0.015	0.027	0.027			0.012	0.012
95%	0.031	0.032					0.016	0.016	0.029	0.029			0.014	0.014
RP			0.597	0.594			0.504	0.504			0.578	0.575	0.5	0.5
5%			0.569	0.564			0.499	0.499			0.554	0.55	0.497	0.496
95%			0.625	0.622			0.509	0.509			0.601	0.598	0.504	0.504
Exp. Loss					0.463	0.462			0.435	0.434	0.442	0.441	0.431	0.431
5%					0.454	0.451			0.432	0.431	0.438	0.436	0.428	0.428
95%					0.474	0.473			0.437	0.436	0.446	0.446	0.434	0.433
Time FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
R^2	0.285	0.285	0.545	0.548	0.388	0.397	0.605	0.606	0.624	0.625	0.897	0.899	0.939	0.94
Panel B: Realized Disasters														
	(1)	(1)	(2)	(2)	(3)	(3)	(4)	(4)	(5)	(5)	(6)	(6)	(7)	(7)
Constant	-0.006	0.008	0.011	0.014	0.012	0.012	-0.003	0.01	-0.001	-0.003	0.005	0.005	0	-0.001
5%	-0.017	-0.002	0.004	0.005	0.005	0.005	-0.008	0	-0.003	-0.007	0.002	0.002	-0.001	-0.002
95%	-0.001	0.025	0.021	0.027	0.017	0.017	0	0.029	0	-0.001	0.007	0.007	0	0
ML	0.064	0.041					0.027	0.01	0.024	0.027			0.011	0.012
5%	0.031	0.032					0.016	-0.006	0.018	0.023			0.008	0.01
95%	0.12	0.06					0.041	0.017	0.028	0.029			0.013	0.013
RP			1.272	1.058			1.158	1.021			0.565	0.557	0.523	0.512
5%			0.606	0.594			0.537	0.531			0.546	0.541	0.506	0.503
95%			2.54	1.936			2.367	1.932			0.589	0.584	0.546	0.524
Exp. Loss					0.493	0.5			0.479	0.491	0.453	0.457	0.45	0.456
5%					0.466	0.469			0.446	0.452	0.437	0.44	0.431	0.436
95%					0.507	0.513			0.496	0.509	0.46	0.463	0.458	0.464
Time FE	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y	N	Y
R^2	0.119	0.392	0.338	0.572	0.863	0.871	0.358	0.58	0.897	0.904	0.972	0.974	0.978	0.98

Table 8: Decompose Next-Period Credit Spread

This table reports the decomposition of the cross-sectional credit spread next period, by regressing credit spreads on market leverage (ML), the risk premium (RP), and the expected loss in default (Exp. Loss) using simulated data. Confidence intervals follow the coefficients. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The confidence intervals are calculated across simulations. The risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0	0.008	0.01	0	0	0.008	0
5%	0	0.002	0.003	0	0	0.002	0
95%	0	0.015	0.016	0	0	0.015	0
ML	0.03			0.03	0.03		0.03
5%	0.029			0.029	0.029		0.029
95%	0.031			0.031	0.031		0.031
RP		0.172		0		0.173	0
5%		0.113		-0.006		0.114	-0.006
95%		0.225		0.005		0.225	0.006
Exp. Loss			0.003		-0.027	-0.004	-0.027
5%			-0.009		-0.029	-0.012	-0.029
95%			0.014		-0.025	0.003	-0.025
Time FE	Y	Y	Y	Y	Y	Y	Y
R^2	0.272	0.057	0.011	0.272	0.274	0.058	0.274
Panel B: Realized Disasters							
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Constant	0.008	0.037	0.044	0.006	0.015	0.043	0.015
5%	-0.001	0.011	0.013	-0.002	-0.001	0.011	-0.001
95%	0.023	0.085	0.104	0.02	0.049	0.104	0.047
ML	0.042			0.057	0.051		0.055
5%	0.031			0.032	0.031		0.032
95%	0.063			0.108	0.09		0.098
RP		-0.292		-0.5		0.081	-0.125
5%		-1.094		-1.455		0.002	-0.317
95%		0.127		-0.01		0.169	-0.009
Exp. Loss			-0.263		-0.28	-0.27	-0.27
5%			-0.412		-0.421	-0.414	-0.405
95%			0.003		-0.018	-0.001	-0.018
Time FE	Y	Y	Y	Y	Y	Y	Y
R^2	0.39	0.358	0.518	0.43	0.589	0.522	0.591

Table 9: Decompose Other Variables

This table reports the decomposition of the cross-sectional firm variables on the risk premium (RP) and the expected loss in default (Exp. Loss) using simulated data. Standard errors are in the parenthesis. Each column of the table reports the coefficients and R^2 from regressions with different combinations of dependent variables. The coefficients and R^2 are averaged across 5000 simulations. The standard errors are calculated across simulations. The risk premium and the expected loss in default are computed using equations (29) and (30) using a model with one-period bonds. Panel A and Panel B respectively report the results in normal times and times with realized disasters.

Panel A: Normal Times						
Dependent	Market Leverage			Equity Value	Capital	Bond Issue
Constant	0.262	0.308	0.261	2.479	0.969	0.059
	(0.004)	(0.004)	(0.004)	(0.013)	(0.003)	(0.001)
RP	6.001		5.965	-20.174	-4.128	-0.811
	(0.030)		(0.030)	(0.110)	(0.026)	(0.007)
Exp. Loss		1.041	0.809	-2.715	-0.618	-0.008
		(0.006)	(0.004)	(0.014)	(0.003)	(0.001)
R^2	0.179	0.006	0.183	0.140	0.033	0.038
Panel B: Realized Disasters						
Dependent	Market Leverage			Equity Value	Capital	Bond Issue
Constant	0.504	0.545	0.500	1.681	0.844	0.113
	(0.003)	(0.003)	(0.002)	(0.009)	(0.002)	(0.001)
RP	4.301		3.907	-13.308	-3.062	-1.351
	(0.024)		(0.021)	(0.074)	(0.017)	(0.005)
Exp. Loss		0.572	0.296	-0.988	-0.313	-0.059
		(0.003)	(0.002)	(0.007)	(0.001)	(0.000)
R^2	0.132	0.043	0.139	0.121	0.059	0.058

Table 10: Cross-Section Regressions

This table reports the cross-section regression of credit spread and market leverage on firm characteristics. The numbers in the data column are from Gomez and Schmid (2020). The coefficients are averaged across 5000 independent simulations. Under each coefficient, I report the 5% and 95% quantiles of the coefficient, which indicates the confidence interval.

	Data	Credit Spread	Data	Market Leverage
Market Leverage	0.090	0.034		
5%		0.031		
95%		0.038		
Size	-0.002	-0.004	0.009	-0.408
5%		-0.006		-0.665
95%		-0.002		-0.113
Profitability	-0.071	-0.218	-0.286	1.8
5%		-0.349		0.896
95%		-0.063		2.161
R^2		0.354		0.271

Table 11: Collapse Period Regressions

This table reports the cross-section regression of equity return and credit spread on firm characteristics during the collapse period. The collapse period indicates the first period of a realized disaster. he coefficients are averaged across 5000 independent simulations. Under each coefficient, I report the 5% and 95% quantiles of the coefficient, which indicates the confidence interval.

Collapse Period	Equity Return		Credit Spread	
Market Leverage	-0.78	-0.788	0.354	0.395
5%	-0.83	-0.852	0.337	0.347
95%	-0.749	-0.755	0.376	0.478
Profitability		1.046		-0.278
5%		0.147		-0.671
95%		1.814		0.032
Size		0.033		0.058
5%		-0.013		0.003
95%		0.096		0.182
R^2	0.889	0.904	0.568	0.582

Table 12: Comparative Statics in Normal Times (Firm)

This table reports comparative statics of parameters that govern real and financial friction in normal times, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports the scenarios with more friction: higher investment and divestment adjustment costs (higher c^+ and c^-), higher fixed and proportional equity issue costs (higher λ_0 and λ_1), higher fixed and proportional bond issue costs (higher ϕ_0 and ϕ_1), less recovery rate (lower ξ), and higher fixed operation cost (higher f). Panel B reports the reverse scenarios with less friction. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters

Panel A: More Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.6	3	0.07	0.06	0.04	0.09	0.6	0.005
1 Year Default Rate	0.656	0.604	0.505	0.44	0.649	0.422	0.517	0.68	1.975
5 Years Default Rate	3.341	3.146	2.585	2.326	3.345	2.231	2.708	3.351	9.631
Credit Spread	92.792	38.019	32.732	18.385	70.114	19.737	35.535	348.147	150.818
Market Leverage	0.311	0.122	0.229	0.051	0.189	0.048	0.098	0.881	0.408
Panel B: Less Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.2	2.55	0.05	0	0.02	0.07	1	0.001
1 Year Default Rate	0.656	0.983	1.248	1.203	1.203	0.683	1.62	0.553	0.411
5 Years Default Rate	3.341	4.983	6.166	5.866	5.963	3.372	7.855	2.889	2.095
Credit Spread	92.792	93.512	164.798	256.315	161.603	222.924	225.47	23.19	74.641
Market Leverage	0.311	0.212	0.524	0.864	0.519	0.929	0.759	0.071	0.233

Table 13: Comparative Statics with Realized Disasters (Firm)

This table reports comparative statics of parameters that govern real and financial friction in samples with realized disasters, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports the scenarios with more friction: higher investment and divestment adjustment costs (higher c^+ and c^-), higher fixed and proportional equity issue costs (higher λ_0 and λ_1), higher fixed and proportional bond issue costs (higher ϕ_0 and ϕ_1), less recovery rate (lower ξ), and higher fixed operation cost (higher f). Panel B reports the reverse scenarios with less friction. The moments are calculated across 5000 independent simulations which include realized disasters.

Panel A: More Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.6	3	0.07	0.06	0.04	0.09	0.6	0.005
1 Year Default Rate	10.445	11.108	9.140	10.66	10.714	10.838	9.773	11.138	12.175
5 Years Default Rate	25.207	27.411	23.084	27.033	26.293	27.498	25.147	25.531	31.209
Credit Spread	315.148	272.856	175.064	232.268	309.954	243.235	234.899	587.257	388.691
Market Leverage	0.563	0.412	0.468	0.309	0.475	0.309	0.342	0.878	0.644
Panel B: Less Frictions									
	Benchmark	c^+	c^-	λ_0	λ_1	ϕ_0	ϕ_1	ξ	f
		0.2	2.55	0.05	0	0.02	0.07	1	0.001
1 Year Default Rate	10.445	9.809	11.317	11.294	11.206	11.410	11.371	10.827	8.582
5 Years Default Rate	25.207	26.030	27.585	26.923	27.450	26.002	28.253	27.470	22.054
Credit Spread	315.148	284.048	383.353	425.090	370.267	400.790	402.667	198.972	244.477
Market Leverage	0.563	0.423	0.701	0.871	0.687	0.910	0.814	0.337	0.422

Table 14: Comparative Statics in Normal Times (Consumer)

This table reports comparative statics of parameters that govern the stochastic discount factors in normal times, holding all other things fixed. The default rates are in percent. The credit spread is in basis points. The first column reports the simulation results with benchmark parameters. Panel A reports results with higher parameter values of time preference factor (β), risk aversion (γ), relative volatility of aggregate productivity (ϕ), elasticity of intertemporal substitution (ψ), persistence of consumption growth (ρ_g), and consumption growth (σ_g). Panel B reports the results with lower parameter values. All the model moments are calculated across 5000 independent simulations, which do not have realized disasters

Panel A: Higher Values							
	Benchmark	β	γ	ϕ	ψ	ρ_g	σ_g
		0.96	7.7	1.05	2.2	0.4	0.01
1 Year Default Rate	0.656	0	2.541	2.604	1.187	0.703	1.154
5 Years Default Rate	3.341	0	12.092	12.367	5.946	3.602	5.783
Credit Spread	92.792	111.142	295.675	257.199	41.725	62.43	61.929
Market Leverage	0.311	0.706	0.823	0.786	0.104	0.203	0.188
Panel B: Lower Values							
	Benchmark	β	γ	ϕ	ψ	ρ_g	σ_g
		0.95	7	0.95	1.8	0.2	0.002
1 Year Default Rate	0.656	7.349	0.557	0.351	0.484	0.761	0.564
5 Years Default Rate	3.341	31.735	2.897	1.823	2.436	3.852	2.852
Credit Spread	92.792	63.886	37.185	30.673	128.719	107.31	130.235
Market Leverage	0.311	0.053	0.126	0.105	0.494	0.358	0.43

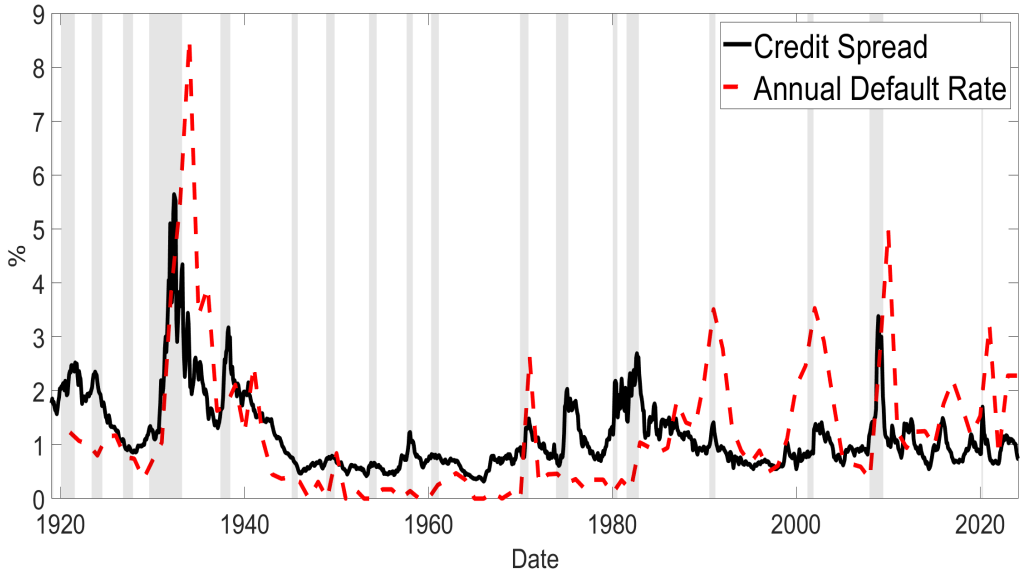
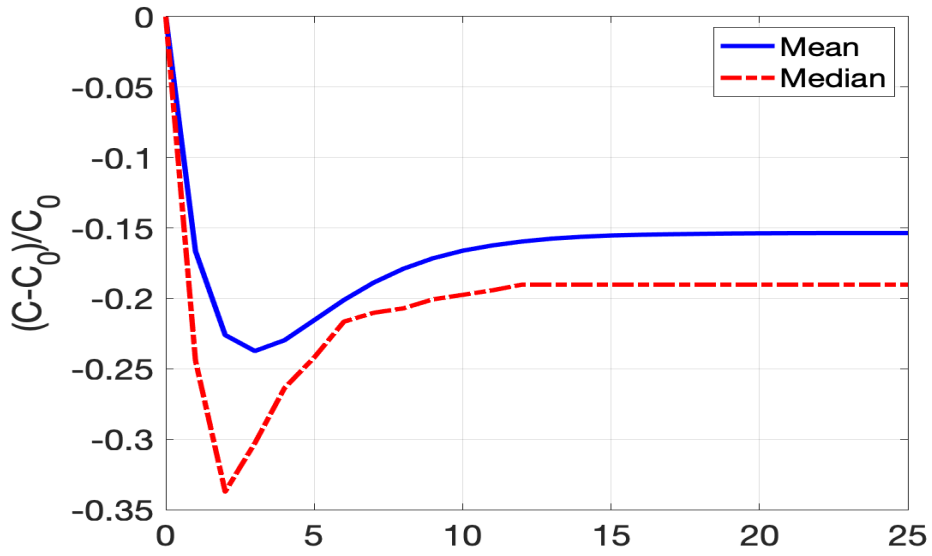
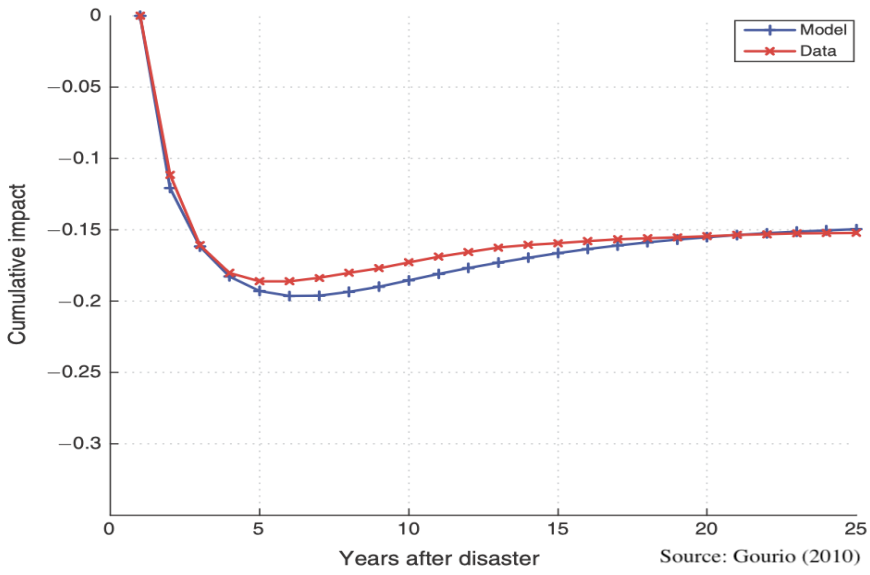


Figure 1: Credit spread and default rate over the business cycle. It plots Moody's annual corporate default rates and the FRED Baa-Aaa credit spreads from 1920 to 2023.

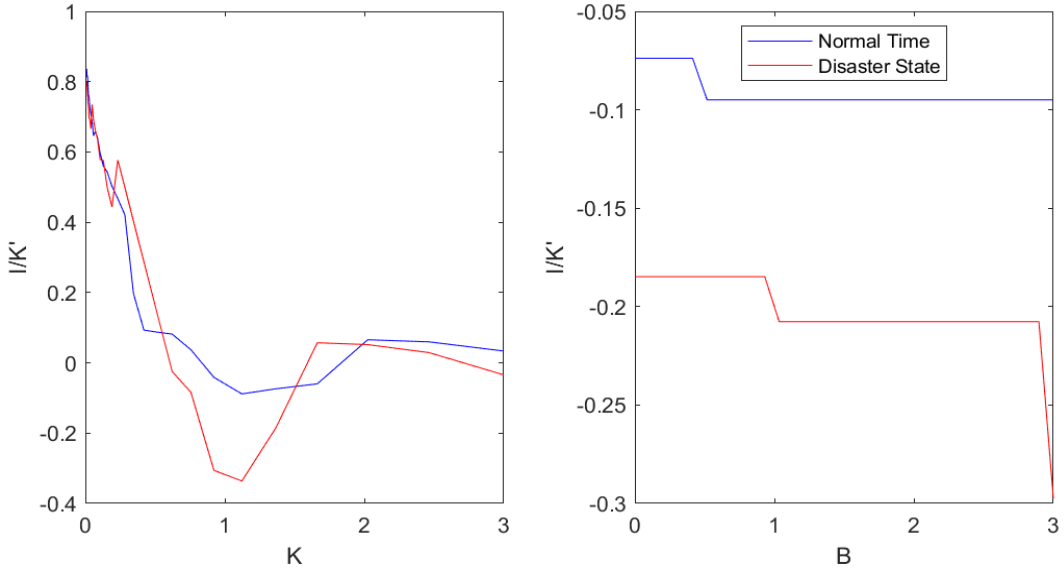


(a) Cumulative consumption growth from my model

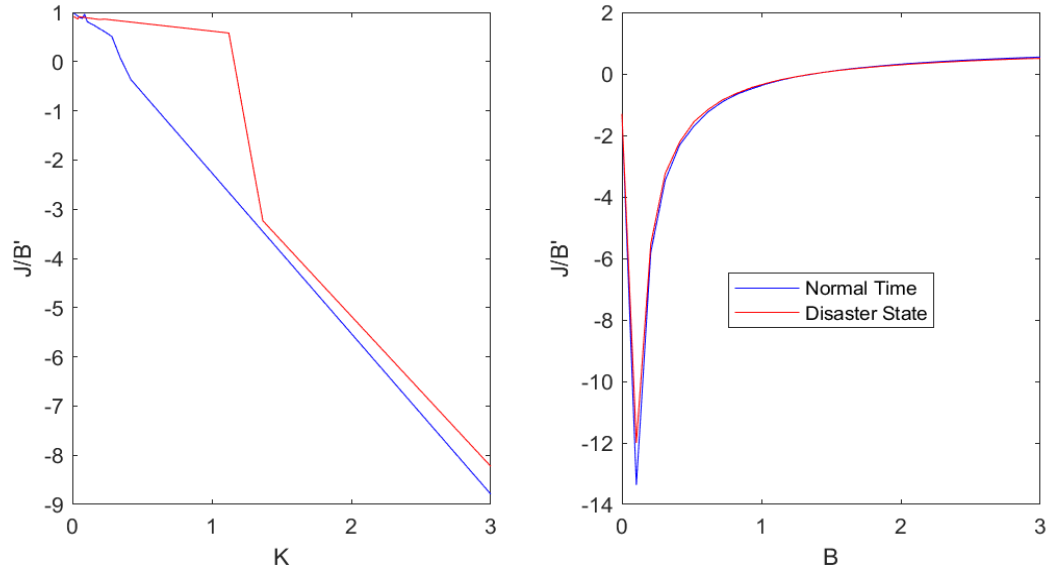


(b) Cumulative consumption growth from Gourio (2012)

Figure 2: The impulse response of consumption to a disaster shock in the model. Panel (a) shows dynamics from my model. In simulations, I calculate the cumulative fractional drop in consumption for 25 years after the economy enters a disaster state. The impulse response is aggregated from the quarterly to annual frequency, and calculated across 10000 simulations. The blue solid line is the mean impulse response, and the red dotted line is the median. Panel (b) shows the dynamics from Gourio (2012).

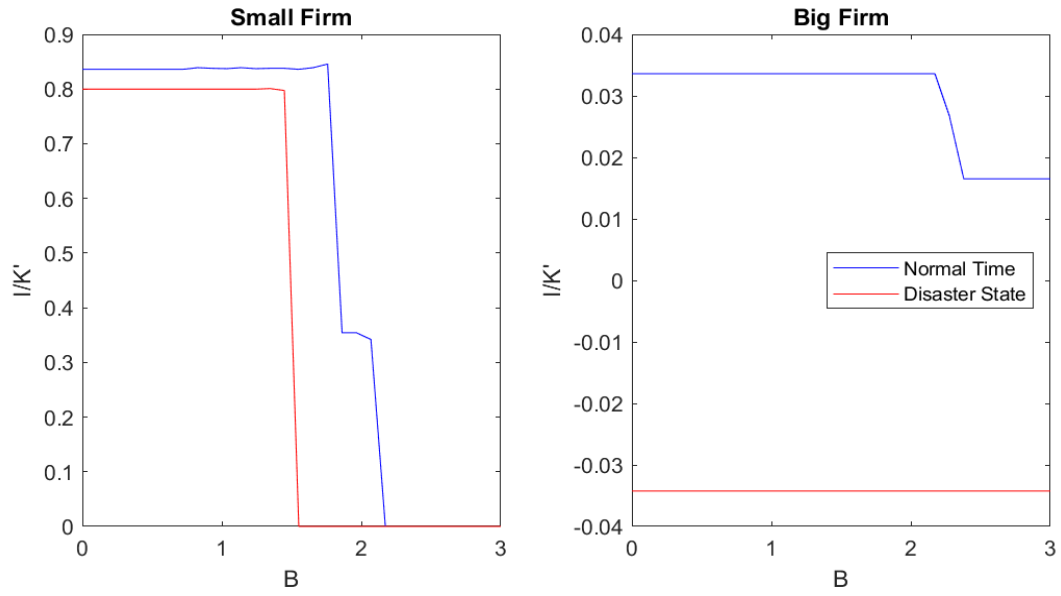


(a) Investment to Capital

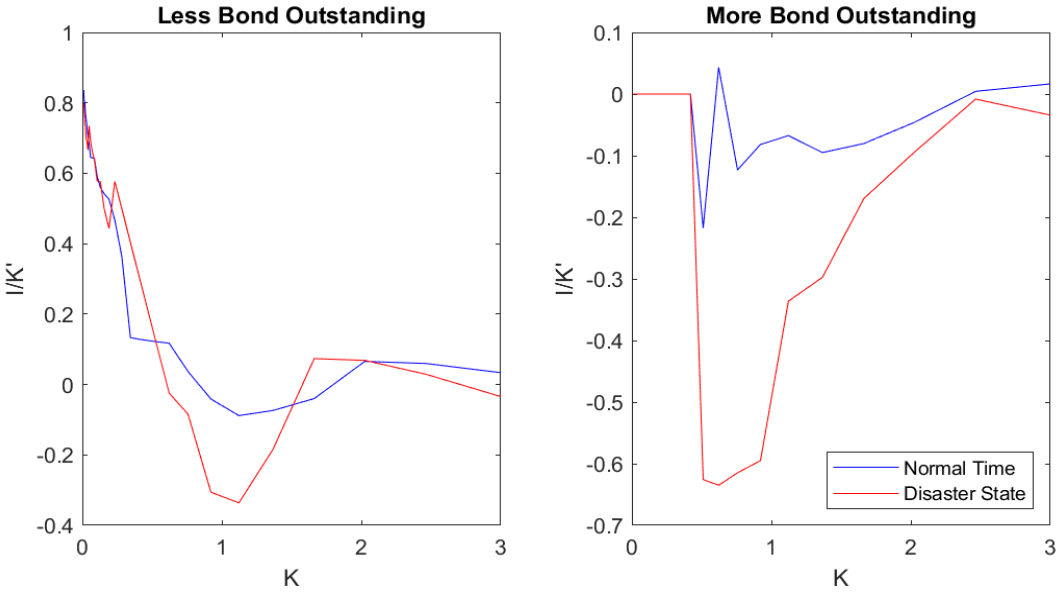


(b) Bond issue to bond outstanding

Figure 3: Policy function at average capital (K) or bond outstanding (B). The upper left figure plots investment-to-capital against K, holding B fixed at the average in the simulation. The upper left figure plots investment-to-capital against B, holding K fixed at the average in the simulation. The lower left figure plots bond issue to bond outstanding against K. The lower right figure plots bond issue to bond outstanding against B. The red line indicates the disaster state. The blue line indicates the average state in normal times.

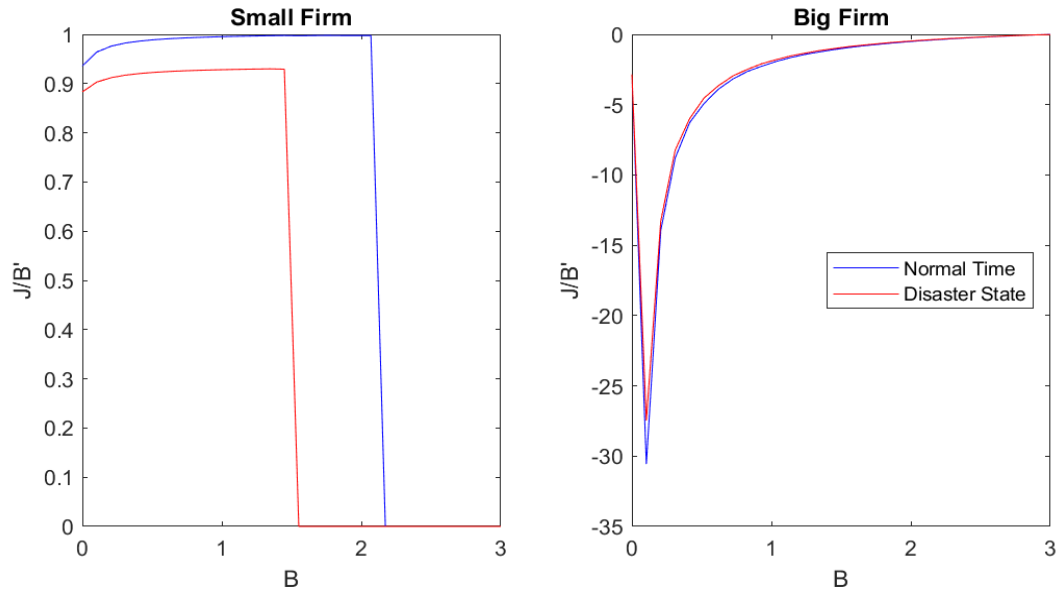


(a) Capital Size

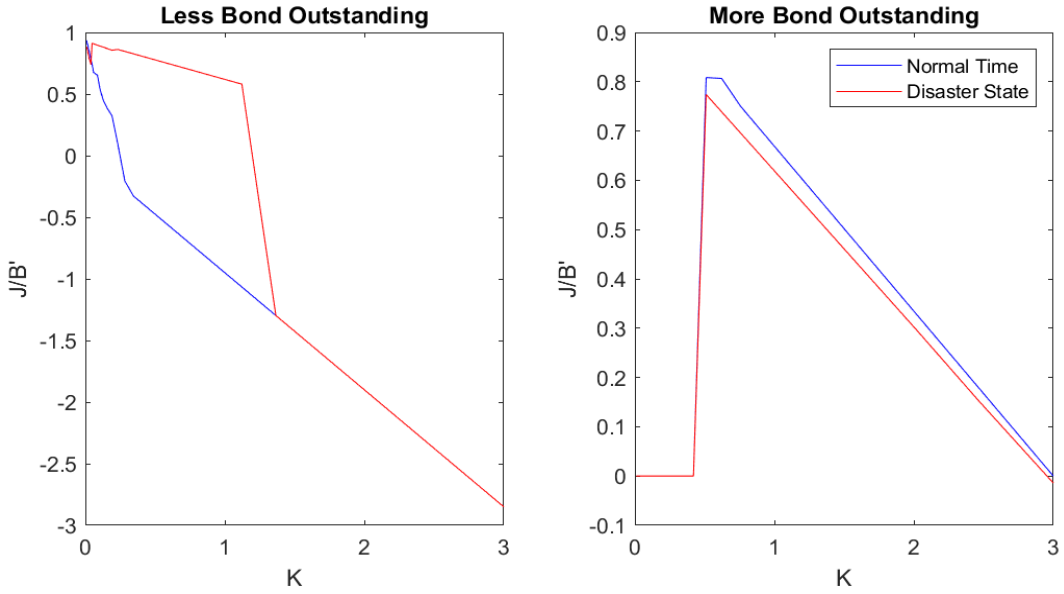


(b) Bond Outstanding

Figure 4: Investment policy function against capital (K) or bond outstanding (B). The upper figures plot investment-to-capital against B, holding K at minimum (left) or maximum value (right). The lower figures plot investment-to-capital against K, holding B at minimum (left) or maximum value (right). The red line indicates the disaster state. The blue line indicates the average state in normal times.

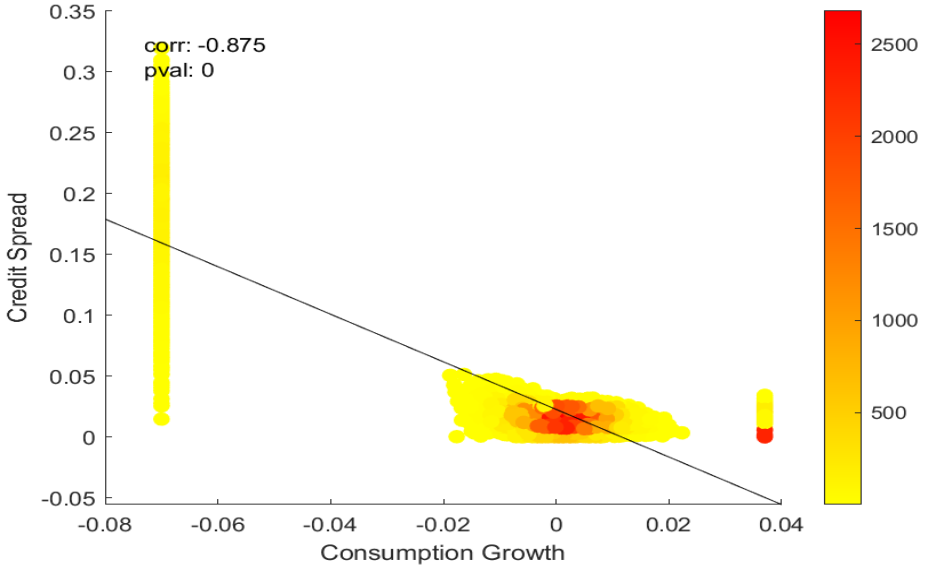


(a) Capital Size

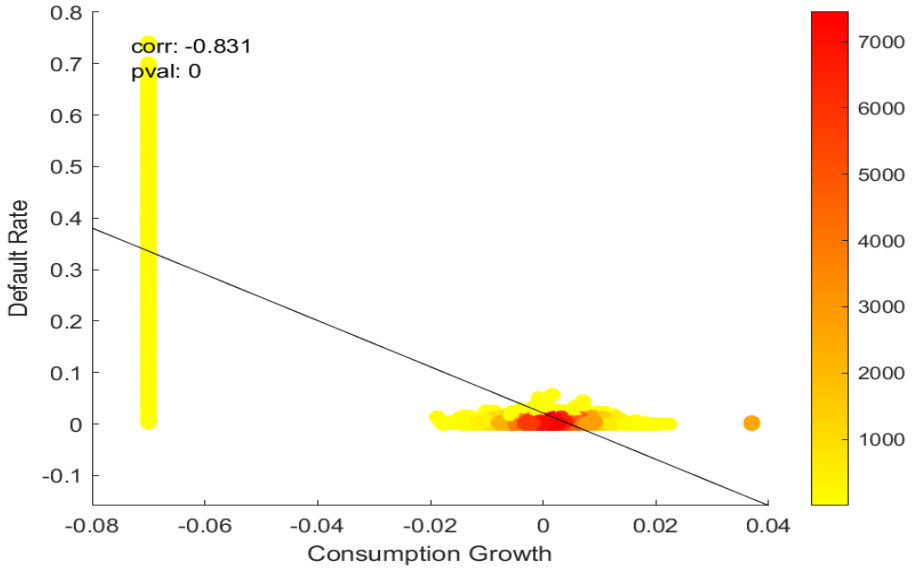


(b) Bond Outstanding

Figure 5: Bond issue policy function against capital (K) or bond outstanding (B). The upper figures plot bond issue to outstanding against B, holding K at minimum (left) or maximum value (right). The lower figures plot bond issue to outstanding against K, holding B at minimum (left) or maximum value (right). The red line indicates the disaster state. The blue line indicates the average state in normal times.

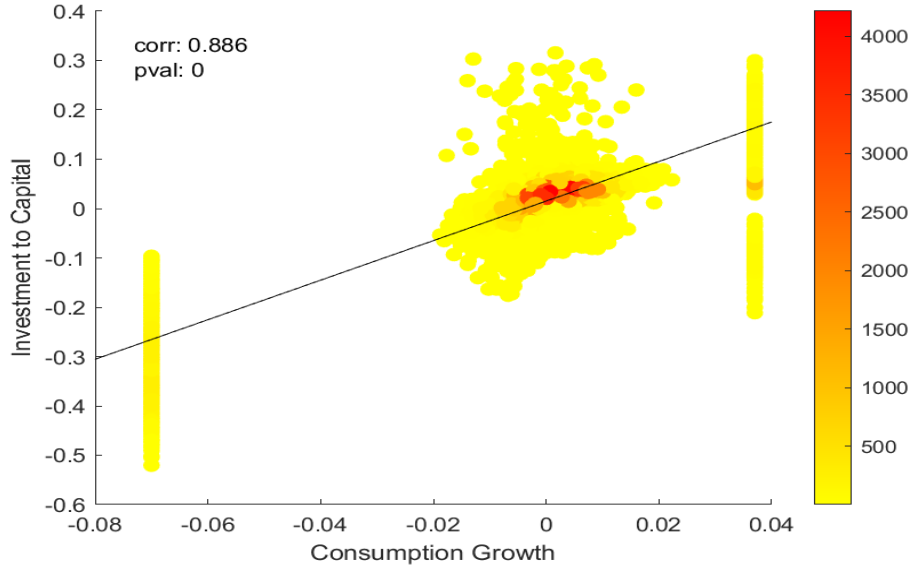


(a) Credit Spread

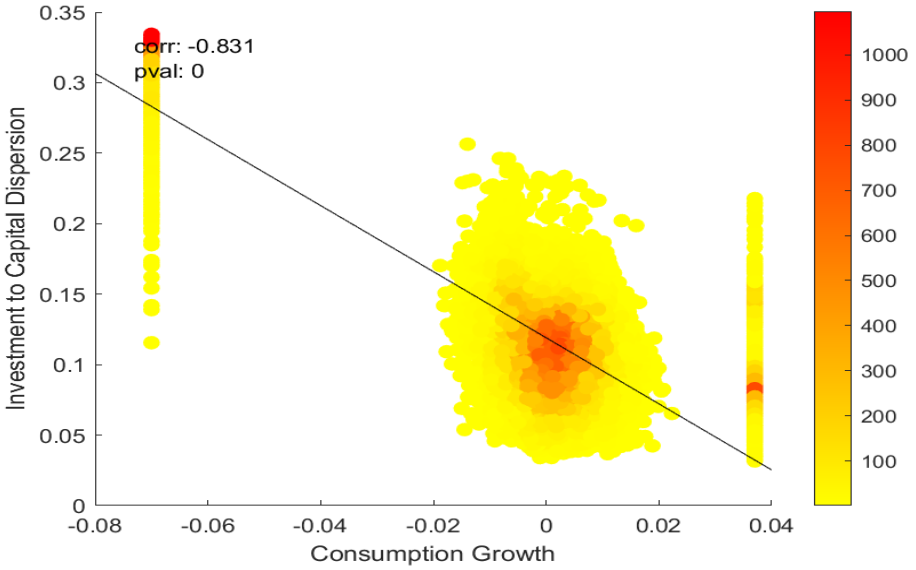


(b) Default Rate

Figure 6: Heatmaps of average cross-section credit spread and default rate against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average credit spread. Panel B shows the average default rate. The correlation and its p-value are reported on the left top corner. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. In each heatmap, red indicates high density, whereas yellow indicates low density.

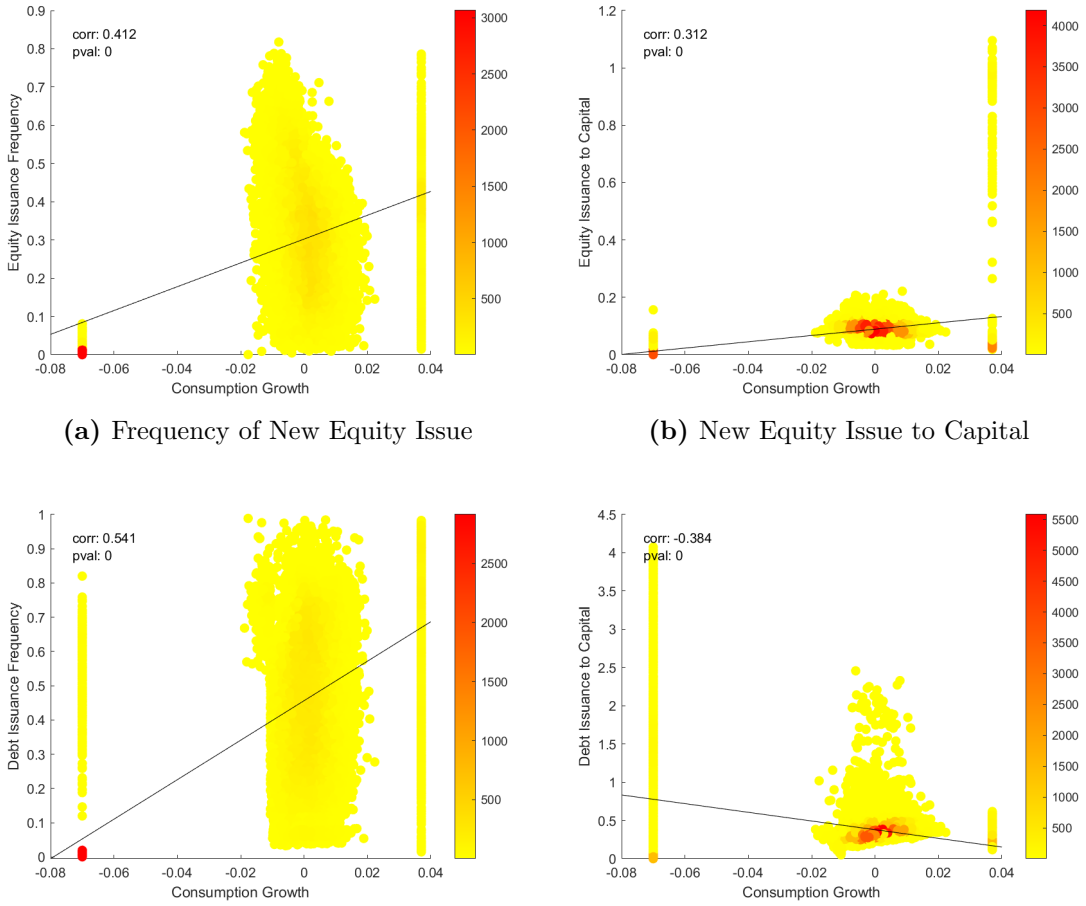


(a) Investment to Capital Rate



(b) Volatility of Investment to Capital Rate

Figure 7: Heatmaps of average and volatility of cross-section investment-to-capital ratio against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average investment-to-capital ratio. Panel B shows the volatility of the investment-to-capital ratio. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.



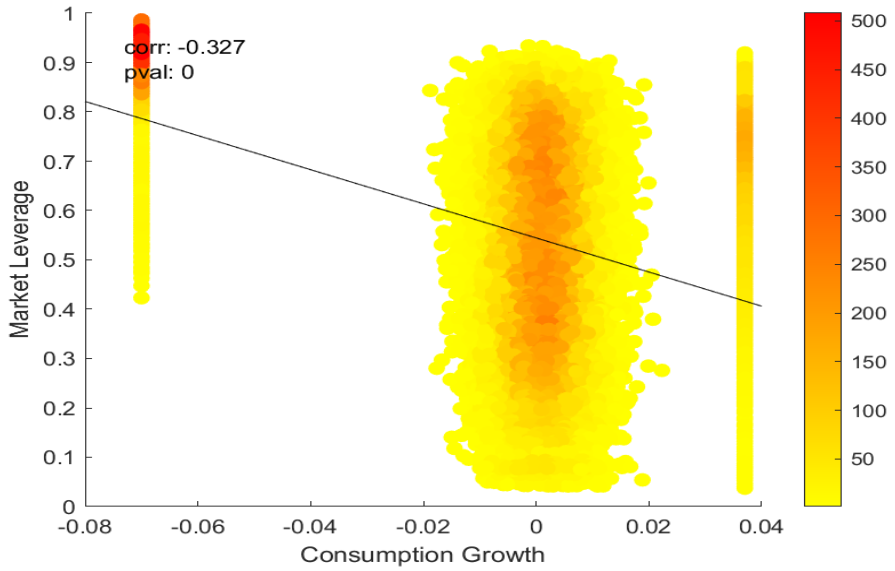
(a) Frequency of New Equity Issue

(b) New Equity Issue to Capital

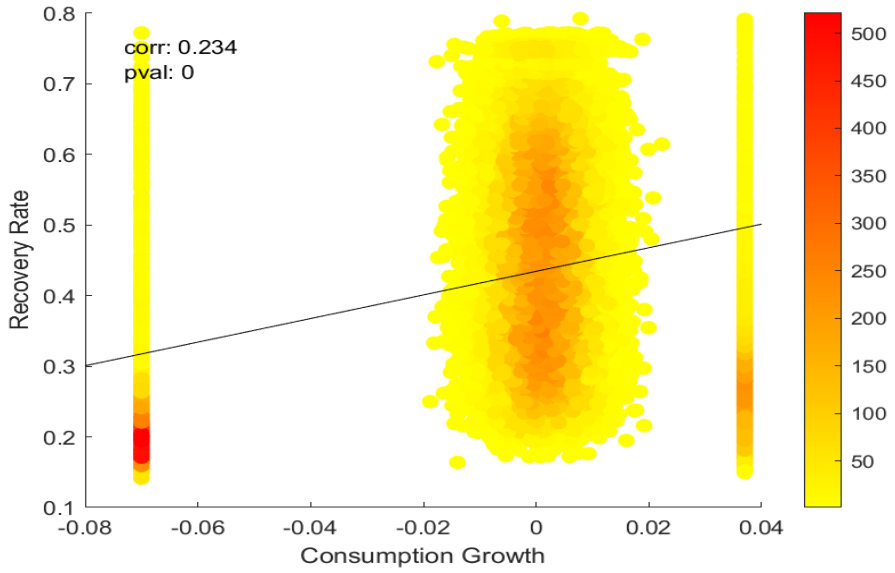
(c) Frequency of New Bond Issue

(d) New Bond Issue to Capital

Figure 8: Heatmaps of bond and equity financing against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the frequency of new equity issues. Panel B shows the amount of new equity issues as a portion of capital. Panel C shows the frequency of new bond issues. Panel B shows the amount of new bond issues as a portion of capital. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.

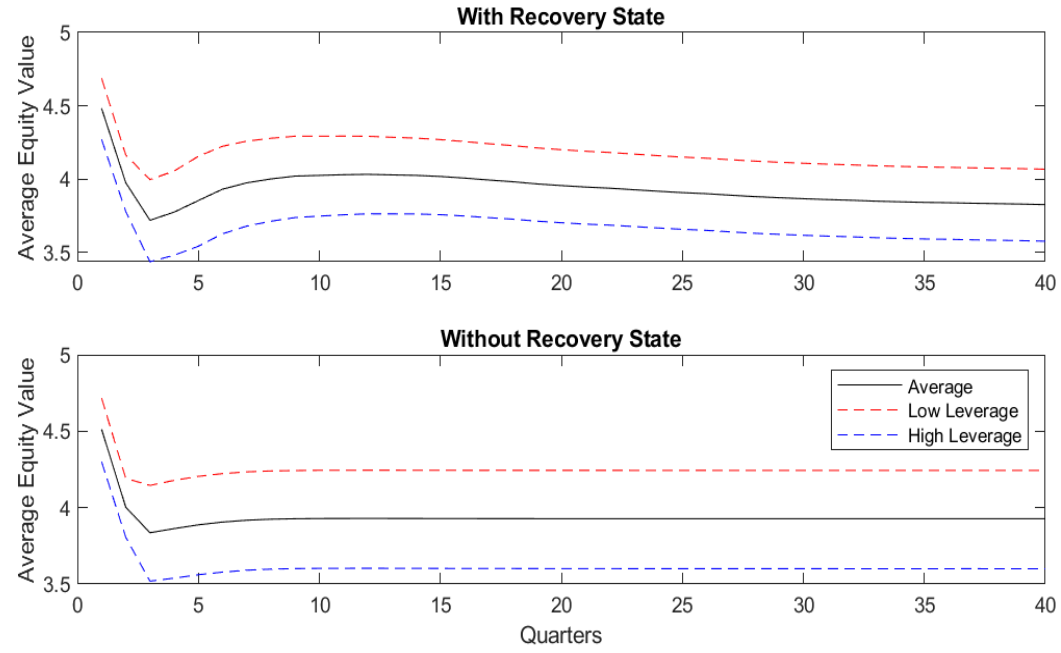


(a) Market Leverage

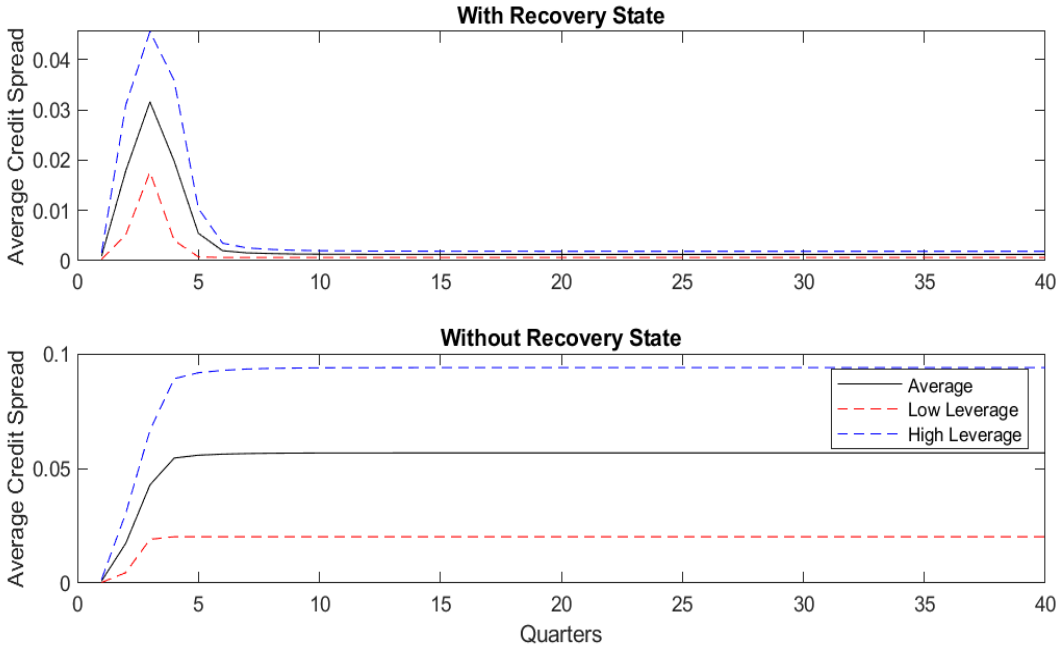


(b) Recovery Rate

Figure 9: Heatmaps of average market leverage and recovery rate against the consumption growth. Plots are based on a long simulation with 50000 quarters. Panel A shows the average market leverage. Panel B shows the recovery rate. In each plot, a regression line is fitted against the consumption growth to show the cyclicality. The correlation and its p-value are reported on the left top corner. In each heatmap, red indicates high density, whereas yellow indicates low density.



(a) Equity Value



(b) Credit Spread

Figure 10: Impulse response functions of equity value and credit spread after a disaster hits. The black line indicates the average across all survival firms. The red dashed line indicates the average across survival firms with lower market leverage. The blue dashed line indicates the average across survival firms with higher market leverage. The upper figures in Panel A and Panel B include recovery states following the disaster states. The lower figures include only disaster states.

A Numerical Solution

To use the value function iteration method to solve the model, I first detrend the original model to a stationary model without growth. I follow the standard method in the macroeconomics literature to use last-period aggregate productivity, X_{t-1}, to deflate capital and bonds. Deflated capital and bonds are denoted as $\widehat{K}_{it} = \frac{K_{it}}{X_{t-1}}$ and $\widehat{B}_{it} = \frac{B_{it}}{X_{t-1}}$, respectively. Consumption growth, g_t, replaces X_t as another state variable because productivity growth g_{xt} is a linear function of g_t. The deflated post-default value function is denoted as $\widehat{V}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t, m_{it}) = \frac{V(Z_{it}, K_{it}, B_{it}, X_t, m_{it})}{X_{t-1}}$. I add a transitory income shock, $m_{it} \overset{i.i.d.}{\sim} \mathcal{N}(0, 0.001)$, as a state variable to overcome the local oscillations.

Other model elements are deflated and denoted in a similar way. Below, I summarize the stationary model that will be solved by grid search

$$\widehat{V}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t, m_{it}) = \max\{0, \widehat{S}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t, m_{it})\} \tag{A.1}$$

$$\widehat{S}(Z_{it}, \widehat{K}_{it}, \widehat{B}_{it}, g_t, m_{it}) = \max_{\widehat{I}_{it}, \widehat{B}_{it+1}} \Big\{ \widehat{e}_{it} + \Lambda(\widehat{e}_{it}) + E_t\big[M_{t+1} \widehat{V}(Z_{it+1}, \widehat{K}_{it+1}, \widehat{B}_{it+1}, g_{t+1}, m_{it+1})\big] exp(\bar{g} + \phi g_t) \Big\} \tag{A.2}$$

subject to (A.3)

$$\widehat{K}_{it+1} exp(\bar{g} + \phi g_t) = (1 - \delta)\widehat{K}_{it} + \widehat{I}_{it} \tag{A.4}$$

$$\widehat{B}_{it+1} exp(\bar{g} + \phi g_t) = (1 - \kappa)\widehat{B}_{it} + \widehat{J}_{it} \tag{A.5}$$

The bond pricing equation has the following form:

$$Q_{it} = E_t\big[M_{t+1}\big((1 - \mathbb{1}_{\widehat{S}_{it+1}<0})(c + \kappa + (1 - \kappa)Q_{it+1}) + \mathbb{1}_{\widehat{S}_{it+1}<0}\Xi_{it+1}\big)\big] \tag{A.6}$$

where the recovery rate is not changed

$$R_{it+1} = \xi \cdot \min\{1, \frac{\Pi_{it+1} + (1 - \delta)K_{it+1}}{B_{it+1}}\} = \xi \cdot \min\{1, \frac{\widehat{\Pi}_{it+1} + (1 - \delta)\widehat{K}_{it+1}}{\widehat{B}_{it+1}}\} \tag{A.7}$$

and the default decision depends on the detrended continuation value

$$\mathbb{1}^D_{it+1} = \mathbb{1}_{\widehat{S}_{it+1}<0} \tag{A.8}$$

The model is solved using value function iteration. Both the current states of capital and bonds, K_{it} and B_{it}, are discretized into 30 grid points. I use the Rowenhorst method to discretize aggregate consumption growth, g_t, into 7 grid points, including 5 normal states, 1 disaster state, and 1 recovery state. Similarly, I discretize idiosyncratic productivity, z_{it}, into 9 states, and the transitory income shock, m_{it}, into 5 states using a truncated Gaussian distribution. To obtain more accurate model solutions, I use finer grids for the decision variables, K_{it+1} and B_{it+1}, which are discretized into 300 grids.

Due to the finer grid for decision variables, the bond pricing function is also defined on finer grids. Consequently, I need to interpolate the value function on these finer grids. However, the post-default value function, V_{it}, has a kink at zero, leading to numerical errors during interpolations. To address this issue and prevent convergence problems, I instead interpolate on the pre-default value function, S_{it}, which is a smoother surface. I then obtain the post-default value function based on the interpolated S_{it}, reducing numerical errors and helping the model converge. Additionally, I include a transitory income shock following the approach in Chatterjee and Eyigungor (2012), which helps in solving multiple local optimum problems and prevents oscillations during the solution process.

To solve the model efficiently, I follow the procedure in Kuehn and Schmid (2014). I start by solving the model with a one-period bond and use its solution as the initial point for solving the model with a two-year bond, and so forth. For tractability, I solve the model with two- and four-year bonds. The convergence threshold for both the value function and bond pricing function is set at 10^{-4}, and in most cases, the value function converges to a level of 10^{-7}. However, note that these tips do not guarantee convergence, as it also depends on the parameter values. To ensure convergence, I experiment with a wide range of parameter values.